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# New superfields for $N$ supersymmetry with central charges

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**Abstract.** We show that a judicious choice of nonlinear realisation of centrally extended $N$ supersymmetry permits the construction of chiral-like superfields with a $2N$-dimensional $a$-number parameter, at the expense of an external 'superspin' index which labels multiplets of several irreducible representations of the Lorentz group. The generalised superfields considered here have $4^{N+[N/2]}$ component fields altogether, as opposed to the $16^N$ expected conventionally. The $N=2$ case is examined in detail, and the decomposition of arbitrary spin superfields into irreducible parts is given. The connection with other $N=2$ superfield analyses is pointed out, and the structure of 'spin-reducing' representations with $p^2+|Z|^2=0$ is exhibited.

## 1. Introduction and main results

This paper is a contribution to the study of $N$-extended supersymmetry in superspace. Although the ultimate goals (see below) of the superfield programme are the construction of realistic interacting models with a view to their quantum behaviour (using superspace techniques to handle the inevitable 'miraculous' cancellations between bosonic and fermionic channels), we remain here at the non-interacting level. Specifically, we introduce new representations which generalise, to the case of $N$-extended supersymmetry with unrestricted central changes, the notion of chiral superfields—a step which general arguments from the usual superfield framework would indicate as problematical (see below). To the extent that a puralistic attack is needed on unresolved questions of the 'holy grails' of maximally extended $N=4$ super Yang-Mills and $N=8$ supergravity models (see, for example, van Nieuwenhuizen 1981 and Milewski 1983a, b), the present work and extensions of it may find application alongside other approaches. Thus, although rapid progress has been made recently in component formalisms at the classical level (see, for example, Duff *et al* 1984), comprehensive results with the quantised models will require full local and covariant superspace techniques. The complexities of the latter have engendered such modifications as $N$ supersymmetry in an $N=1$-'component superfield' basis (Fayet 1976, 1979, Gates 1981, Milewski 1983a, b), or light-cone formalisms (Mandelstam 1982, Brink *et al* 1983a, b, Namazie *et al* 1983, Taylor 1983), both of which necessitate some sacrifice (auxiliary field content, and manifest Lorentz invariance and locality, respectively). There are indications based on counting arguments that beyond $N=2$ the full $N$ superspace is intrinsically inadequate to represent physical multiplets (Rivelles and Taylor 1981, Rocek and Siegel 1981), unless particular 'spin-reducing' representations are used (Fayet 1976, 1979, Sohnius 1978, Rands and Taylor 1983a, b). These emerge naturally in the present work (see below).

 

The method of nonlinear realisations is conventionally applied to $N$ supersymmetry by considering functions on coset spaces $(Z\times \mathrm{O}(3,1)\times \mathrm{K})\,Ⓢ\,T_{4/4N}/\mathrm{O}(3,1)\times \mathrm{K}$, where $\mathrm{O}(3,1)$ is the Lorentz group, K is an internal symmetry group (a subgroup of $\mathrm{Sp}(2N)$), $T_{4/4N}$ is the nilpotent algebra of translations and supertranslations, and $Z\times$ denotes the Abelian central charges (see, for example, Salam and Strathdee 1978, Wess and Bagger 1982, Gates *et al* 1983). One then has superfields $\Phi^{\{\lambda\}}_{(p,q)}(x^\mu, \theta^{ai}, \theta_{ai}, \ldots)$, functions of spinor parameters transforming as $(\frac{1}{2},0)\times\{1\}+(0,\frac{1}{2})\times\{1\}$ under $\mathrm{O}(3,1)\times$ K, plus the usual Minkowski-space coordinates $x^\mu$, and some additional bosonic central charge coordinates; the superfields possess external $\mathrm{O}(3,1)\times \mathrm{K}$ transformation properties in some representation $(p,q)\times\{\lambda\}$.

These superfields and their corresponding physical states have been analysed recently (Sokatchev 1975, 1981, Rittenberg and Sokatchev 1981, Siegel and Gates 1981, Ferrara *et al* 1981, Ferrara and Savoy 1982, Taylor 1980, Pickup and Taylor 1981, Lopuszański and Wolf 1982, Kim 1984). Superfield expansions in the $4N$ Grassmann coordinates involve $2^{4N}=16^N$ components, and a satisfactory analysis requires the use of the maximal automorphism symmetry of the algebra ($\mathrm{Sp}(2N)$ in the absence of central charges). The superfield differential realisations are in fact irreducible only with respect to an enlarged superalgebra containing generators (so-called 'covariant derivatives') which anticommute with the supertranslations. Labelling operators, including Casimir invariants, and corresponding projectors on the irreducible superfields can thus be constructed in terms of these. A useful set of projectors corresponds to the 'chiral' case where a superfield is constrained to have a vanishing covariant derivative, and consequently can be solved in terms of a function only of $x^\mu+\mathrm{i}\theta\sigma^\mu\bar{\theta}$, and say $\theta_{ai}$, thereby having only $2^{2N}=4^N$ components. However, since central charges arise from the anticommutation of covariant derivatives, care must be exercised lest on-shell conditions (e.g. $p^2=0=|Z|^2$) already be applied as constraints (see especially Lindgren 1982, Restuccia and Taylor 1983).

The approach expounded here differs in two fundamental respects from the conventional procedure. Firstly, the central charges are realised as multiplicative, complex parameters rather than extra coordinates. Secondly, the superfields are functions of Grassmann parameters of only a particular chirality but take their values in a graded representation space of a superalgebra. Thus superfields are functions on the coset space $(Z\times \mathrm{O}(3,1)\times \mathrm{K})\,Ⓢ\,T_{4/4N}/(Z\times \mathrm{O}(3,1)\times \mathrm{K})\,Ⓢ\,T_{0/2N}$, where $T_{0/2N}$ is the superalgebra of supertranslations of a particularly chirality. These superfields are functions of only $2N$ Grassmann coordinates but possess external 'superspin' corresponding to representations of the graded Lorentz group $(Z\times \mathrm{O}(3,1)\times \mathrm{K})\,Ⓢ\,T_{0/2N}$ (cf Ivanov and Sorin 1980). As we shall see below, these include $2^{2[N/2]}=4^{[N/2]}$ irreducible representations of the Lorentz group, giving a total of $4^{N+[N/2]}$ components.

The $N$-extended Poincaré supersymmetry algebra, $\mathscr{G}$, with $\mathrm{K}=\mathrm{SO}(N)$ and $\frac{1}{2}N(N-1)$ complex central charges, $Z_{ij}$, where $i,j=1,\ldots,N$, is

$$
\begin{aligned}
&[J_{\mu\nu}, P_\rho]=\mathrm{i}(\eta_{\nu\rho}P_\mu-\eta_{\mu\rho}P_\nu)\\
&[J_{\mu\nu}, J_{\rho\sigma}]=\mathrm{i}(\eta_{\mu\rho}J_{\mu\sigma}-\eta_{\mu\rho}J_{\nu\sigma}+\eta_{\mu\sigma}J_{\nu\rho}-\eta_{\nu\sigma}J_{\mu\rho})\\
&\{Q_{\alpha i}, \bar{Q}_{\dot{\alpha}j}\}=-2\delta_{ij}(\sigma^\mu)_{\alpha\dot{\alpha}}P_\mu\\
&\{Q_{\alpha i}, Q_{\beta j}\}=2\varepsilon_{\alpha\beta}Z_{ij}\\
&\{\bar{Q}_{\dot{\alpha}i}, \bar{Q}_{\dot{\beta}j}\}=2\varepsilon_{\dot{\alpha}\dot{\beta}}\bar{Z}_{ij}\\
&[J_{\mu\nu}, Q_{\alpha i}]=-\mathrm{i}(\sigma_{\mu\nu})_\alpha{}^\beta Q_{\beta i}
\end{aligned}
\qquad (1)
$$

$$[J_{\mu\nu}, \bar{Q}_{\dot{\alpha}i}] = \mathrm{i}(\bar{\sigma}_{\mu\nu})_{\dot{\alpha}}{}^{\dot{\beta}}\bar{Q}_{\dot{\beta}i}$$

$$[Q_{\alpha i}, T_{jk}] = (t_{jk})_i{}^l Q_{\alpha l}$$

$$[\bar{Q}_{\dot{\alpha}i}, T_{jk}] = (t_{jk})_i{}^l \bar{Q}_{\dot{\alpha}l}$$

$$[T_{ij}, T_{kl}] = C^{mn}_{ij,kl} T_{mn}.$$

Here $P_\mu$ and $J_{\mu\nu}$ generate the Poincaré group and $Q_{\alpha i}$ and $\bar{Q}_{\dot{\alpha}i}$ are the supertranslation generators. $T_{jk}$ are the SO($N$) generators with Hermitian representation $(t_{jk})_i{}^1$ and structure constant $C^{mn}_{ij,kl}$. The metric is $\eta_{\mu\nu} = (-, +, +, +)$ and all other (anti)commutators are zero.

Induced representations of $\mathscr{G}$ are obtained here, by taking superspace as the coset space, G/H, where G is the SO($N$)-extended super-Poincaré group with corresponding superalgebra $\mathscr{G}$, H is a subgroup of G with corresponding superalgebra $\mathscr{H} = \{J_{\mu\nu}, Q_{\alpha i}, T_{ij}, Z_{ij}\}$. This coset space can be parameterised as $\exp[\mathrm{i}(x^\mu P_\mu + \bar{\theta}^{\dot{\alpha}i}\bar{Q}_{\dot{\alpha}i})]$ with coordinates $(x^\mu, \bar{\theta}^{\dot{\alpha}i})$, where $x^\mu(\bar{\theta}^{\dot{\alpha}i})$ is a $c-(a-)$ number parameter). Representations of $\mathscr{G}$ are afforded by superfields $\Phi_A(x^\mu, \bar{\theta}^{\dot{\alpha}i})$ which are functions on G/H taking their values in a representation space, $\mathscr{V}$, of $\mathscr{H}$.

Obtaining representations of $\mathscr{G}$ in this manner presumes that the representations of $\mathscr{H}$ are known. Since $\mathscr{H}$ is also a superalgebra an analogous procedure to the above is followed. Firstly, we note that with respect to the positive, negative and zero roots of SO($N$) the generators $T_{ij}$ and $Q_{\alpha i}$ may be written in bases $T_{ij} = \{T^+_a, T^-_a, T^0_n\}$ and $Q_{\alpha i} = \{Q^+_{\alpha n}, Q^-_{\alpha n}, Q^0_\alpha\}$ respectively, where $n = 1, \ldots, [\frac{1}{2}N]$, $a = 1, \ldots, [\frac{1}{2}N][\frac{1}{2}(N-1)]$ and $Q^0_\alpha$ only exists for $N$ odd, for which $[T^\pm, T^0] \subset T^\pm$, $[T^+, T^-] \subset T^0$, $\{Q^+, Q^-\} \subset Z$, $[T^\pm, Q^\mp] \subset Q^\pm$ and all other (anti)commutators involving these generators are zero. To implement the inducing construction on $\mathscr{H}$, a subgroup, H′, of H is chosen with corresponding superalgebra $\mathscr{H}' = \{J_{\mu\nu}, T^+_a, T^0_n, Q^+_{\alpha n}, Q^0, Z_{ij}\}$. It is possible to decompose $\mathscr{H}'$ as $= \mathscr{H}'_0 + \mathscr{H}'_+$, where $\mathscr{H}'_+ = \{T^+_a, Q^+_{\alpha n}, Q^0_\alpha\}$ is an ideal. Representations of $\mathscr{H}'_0$ are then extended to $\mathscr{H}$ by taking them to be zero on $\mathscr{H}'_+$. The coset space, H/H′, is parameterised as $\exp[\mathrm{i}(y^a T^-_a + \theta^{\alpha n} Q^-_{\alpha n})]$ with coordinates $(y^a, \theta^{\alpha n})$ and respresentations of $\mathscr{H}$ are afforded by superfields $\Psi_B(y^a, \theta^{\alpha n})$ which are functions on this coset space taking their values in a representation space of $\mathscr{H}'$. By considering the group action on the coset representatives a differential representation of the generators of $\mathscr{H}$ can be obtained and their action on superfields examined to determine the finite-dimensional, irreducible representations of $\mathscr{H}$.

Since an expansion of $\Psi_B(y^a, \theta^{\alpha n})$ in $\theta^{\alpha n}$ yields $2^{2[N/2]}$ component fields, while an expansion of $\Phi_A(x^\mu, \bar{\theta}^{\dot{\alpha}i})$ in $\bar{\theta}^{\dot{\alpha}i}$ yields $2^{2N}$ component fields, each of which carries a representation of $\mathscr{H}$, there are a total of $4^{N+[N/2]}$ component fields. There may be, however, fewer than this if the representation of $\mathscr{H}$ carried by $\Psi_B$ is reducible.

The problem of determining irreducible representations of $\mathscr{G}$ must now be addressed. In the conventional procedure, discussed earlier, the algebra $\mathscr{G}$ is extended to include covariant derivatives which, together with the generators of $\mathscr{G}$, provide a basis in superspace for the enveloping algebra, under which the superfields are still invariant. Since the superfields provide a representation space for the extended algebra they are expected to be reducible under $\mathscr{G}$. A similar situation exists in the present case.

The differential form of the spinorial generators is

$$\bar{Q}_{\dot{\alpha}i} = -\mathrm{i}\partial/\partial\bar{\theta}^{\dot{\alpha}i} + \mathrm{i}\bar{\theta}^{\dot{\beta}j}\varepsilon_{\dot{\alpha}\dot{\beta}}Z^0_{ij} \tag{2}$$

$$Q_{\alpha i} = 4\bar{\theta}^{\dot{\alpha}i}(\sigma^\mu)_{\alpha\dot{\alpha}}P_\mu - Q^0_{\alpha i} \tag{3}$$

where $Z^0_{ij}$ and $Q^0_{\alpha i}$ are matrix representations of the corresponding generator. Remembering that $Z_{ij}$ is totally antisymmetric, (2) and (3) tell us that a basis for the enveloping algebra in superspace is provided by extending the superalgebra to include a new set of generators, $\bar{S}_{\dot{\alpha} i} = \partial/\partial\bar{\theta}^{\dot{\alpha} i}$. It is noted, however, that the complete set of differential operators, $\partial/\partial\bar{\theta}^{\dot{\alpha} i}$, is not required for this basis if $N$ is odd. This becomes apparent if one regards the second term on the right of (2) as a set of $N$ linear equations in the variables $\bar{\theta}^j$ with coefficients $Z^0_{ij}$. Since $Z^0_{ij}$ is a totally antisymmetric $N \times N$ matrix it will have zero determinant for odd $N$, and consequently the equations will be linearly dependent. Thus, for $N$ odd, at least one of the generators, $\bar{S}_{\dot{\alpha} i}$, can be regarded as being constructed from linear combinations of the other generators. This extended algebra is denoted by $\bar{\mathscr{G}}$. The generators $\bar{S}_{\dot{\alpha} e}$ are, however, significantly different to the covariant derivatives of the conventional procedure in that they do not anticommute with $\bar{Q}_{\dot{\alpha} i}$ and $Q_{\alpha i}$ and thus cannot be used to generate irreducible representations of $\bar{\mathscr{G}}$ from irreducible representations of $\mathscr{G}$.

For the present purposes of analysing superfields into irreducible representations of supersymmetry, we eschew delving into the details of Casimir invariants and projection operators (cf Sokatchev 1975, Rittenberg and Sokatchev 1981, Taylor 1980, Pickup and Taylor 1981, Siegel and Gates 1981, Kim 1984) in favour of a treatment based upon the recognition of highest (and lowest) weight components (cf Kac 1978, Farmer and Jarvis 1984) and explicit construction of the invariant subspaces therefrom. First, it is noted that $\bar{S}_{\dot{\alpha} i}$ and $\bar{Q}_{\dot{\alpha} i}$ may be cast in bases $\bar{S}_{\dot{\alpha} i} = \{\bar{S}^+_{\dot{\alpha} n}, \bar{S}^-_{\dot{\alpha} n}, \bar{S}^0_{\dot{\alpha}}\}$ and $\bar{Q}_{\dot{\alpha} i} = \{\bar{Q}^+_{\dot{\alpha} n}, \bar{Q}^-_{\dot{\alpha} n}, \bar{Q}^0_{\dot{\alpha}}\}$ with similar properties to $Q^{\pm}_{\alpha n}$ and $Q^0_{\alpha}$. From the discussion of the previous paragraph we note that it is possible to regard $\bar{S}^0_{\dot{\alpha}}$ as a linear combination of the other generators of $\bar{\mathscr{G}}$ and thus it is not an independent generator. Consequently in the following work it will not be counted in the explicit construction of states.

Irreducible representations of $\mathscr{G}$ are obtained from an inducing construction by choosing a subalgebra, $\mathscr{H}$, of $\bar{\mathscr{G}}$ where $\mathscr{H} = \{\bar{S}^+_{\dot{\alpha} n}, \bar{Q}^+_{\dot{\alpha} n}, Q^+_{\alpha n}, Q^0_{\alpha}, T^+_a, T^0_n, Z_{ij}, J_{\mu\nu}\}$ and states, $\Lambda$, which are irreducible representations of the little algebra $\mathscr{H}_0 = \{T^0_n, J_{\mu\nu}, Z_{ij}\}$ and which satisfy $\bar{S}^+_{\dot{\alpha} n}\Lambda = \bar{Q}^+_{\dot{\alpha} n}\Lambda = Q^+_{\alpha n}\Lambda = Q^0_{\alpha}\Lambda = T^+_a\Lambda = 0$. This last requirement is justified from the fact that $\mathscr{H}^+ = \{\bar{S}^+_{\dot{\alpha} n}, \bar{Q}^+_{\dot{\alpha} n}, Q^+_{\alpha n}, Q^0_{\alpha}, T^+_a\}$ is an ideal of $\mathscr{H}$. A basis for an irreducible representation of $\mathscr{G}$, of states which are representations of $\mathscr{H}$, is obtained by acting with monomials of $\bar{Q}^-_{\dot{\alpha} n}$, $\bar{Q}^0_{\dot{\alpha}}$ and $Q^-_{\alpha n}$ on $\Lambda$. A similar basis, for irreducible representations of $\bar{\mathscr{G}}$, is obtained by acting with monomials of $\bar{Q}^-_{\dot{\alpha} n}$, $\bar{Q}^0_{\dot{\alpha}}$, $Q^-_{\alpha n}$ and $\bar{S}^-_{\dot{\alpha} n}$ on $\Lambda$. Thus, a superfield will possess $2^{2[N/2]}$ irreducible multiplets of $\mathscr{G}$ each of which contains $2^{2N}$ irreducible multiplets of $\mathscr{H}_0$, giving a total of $4^{N+[N/2]}$ component fields as required by the superfield analysis. Unlike the conventional case, where the irreducible multiplets of $\mathscr{G}$ are invariant under the covariant derivatives, the $\bar{S}_{\dot{\alpha} e}$ will mix these representations.

This programme is carried out in detail in § 2 below for SO(2)-extended Poincaré supersymmetry. The general superfield expansion is given in (11), and the highest weight in terms of the above discussion is $F_+$; there is an analogous lowest weight $f_-$. The irreducible superfields constructed from these appear as invariant subspaces, and the remaining irreducible superfields are realised as factor spaces of the general superfield. A slightly more convenient basis for these spaces emerges from the analysis (see tables 2 and 3); the component form of the supertranslation action is given in table 4 for the $F_+$ multiplet. Appendices 1 and 2 provide technical details of the matrix- and projector-formalism for the arbitrary spin superfields, and the $\theta$ calculus, required for § 2.

It has been observed (Fayet 1976, 1979, Sohnius 1978, Rands and Taylor 1983a, b) that for the special cases $p^2+|Z|^2=0$ a constraint can be imposed on the supertranslation generators effecting a drastic reduction in dimension. In our treatment $p^2+|Z|^2=0$ is an atypicality condition under which otherwise irreducible superfields become indecomposable; on each factor space the constraint is implemented modulo coset elements. In the $N=2$ case (see § 2 for details) there are four irreducible factors each with four components.

As mentioned above, it is via these so-called 'spin-reducing' cases (which will become $p^2=|Z|^2=0$ on shell) that one hopes to avoid the 'component explosion', and give a full off-shell formalism for $N=3$ supersymmetry (for the results of a different implementation of this approach, see Davis *et al* 1984). As far as the present work is concerned, we observe that bilinear invariants may always be written down (at least in component form) which in fact serve as definitions of the contragrediently-transforming superfield; presumably a corresponding projector formalism could be found (Taylor 1980, Pickup and Taylor 1981, Bufton and Taylor 1983, Kim 1984). However, in practice such projections are implemented via gauge freedoms and other constraints so there is little to be gained in the absence of these and without interactions. In this connection the possibility of a geometrical framework for the present superfield realisations also raises interesting questions.

## 2. $N=2$ extended supersymmetry with central charge

The SO(2) graded extension of the Poincaré algebra, $\mathscr{G}$, is obtained by taking, in addition to the generators of the Poincaré algebra, $P_\mu$ and $J_{\mu\nu}$, the generator for SO(2) transformations, $T$, and the Majorana spinor charges $Q_{\alpha a}$ and $\bar{Q}_{\dot\alpha a}$, where $1\leqslant\alpha,\ \dot\alpha\leqslant 2$ and $a=+,-$. In its most general form the algebra may also include a central charge, $Z$. In the Weyl representation these generators satisfy the following graded Lie algebra:

$$
\begin{aligned}
&[J_{\mu\nu}, P_\rho]=\mathrm{i}(\eta_{\nu\rho}P_\mu-\eta_{\mu\rho}P_\nu)\\
&[J_{\mu\nu}, J_{\rho\sigma}]=\mathrm{i}(\eta_{\nu\rho}J_{\mu\sigma}-\eta_{\nu\rho}J_{\nu\sigma}+\eta_{\mu\sigma}J_{\nu\rho}-\eta_{\nu\sigma}J_{\mu\rho})\\
&\{Q_{\alpha\pm}, \bar{Q}_{\dot\alpha\mp}\}=-4(\sigma^\mu)_{\alpha\dot\alpha}P_\mu\\
&\{Q_{\alpha a}, Q_{\beta b}\}=4\mathrm{i}\varepsilon_{\alpha\beta}\varepsilon_{ab}Z\\
&\{\bar{Q}_{\dot\alpha a}, \bar{Q}_{\dot\beta b}\}=4\mathrm{i}\varepsilon_{\dot\alpha\dot\beta}\varepsilon_{ab}\bar{Z}\\
&[J_{\mu\nu}, Q_{\alpha a}]=-\mathrm{i}(\sigma_{\mu\nu})_\alpha{}^\beta Q_{\beta a}\\
&[J_{\mu\nu}, \bar{Q}_{\dot\alpha a}]=\mathrm{i}(\bar\sigma_{\mu\nu})_{\dot\alpha}{}^{\dot\beta}\bar{Q}_{\dot\beta a}\\
&[T, Q_{\alpha\pm}]=\pm Q_{\alpha\pm}\\
&[T, \bar{Q}_{\dot\beta\pm}]=\pm\bar{Q}_{\dot\alpha\pm}
\end{aligned}
\tag{4}
$$

where $\varepsilon_{-+}=-\varepsilon_{+-}=+1$ and all other (anti)commutators are zero. The metric is taken as $\eta_{\mu\nu}=(-,+,+,+)$.

Following the procedure discussed in § 1 the subalgebra, $\mathscr{H}$, is taken to be $\mathscr{H}=\{J_{\mu\nu}, T, Z, Q_{\alpha a}\}$ with little group $\mathscr{H}_0=\mathscr{H}$. The cosets $\mathscr{G}/\mathscr{H}$ are labelled by the elements $\exp[\mathrm{i}(x^\mu P_\mu+\bar\theta^{\dot\alpha a}\bar{Q}_{\dot\alpha a})]$ and the superfields are defined as functions, $\Phi_A(x^\mu, \bar\theta^{\dot\alpha a})$, taking their values in a representation space of $\mathscr{H}_0$.

Irreducible representations of $\mathscr{H}$ can be constructed by considering states $|\,\rangle$ carrying labels $(p,q)\times\{t\}$ of $\mathrm{O}(3,1)\times\mathrm{U}(1)$ and such that $Q_{\alpha+}|\,\rangle=0$. The set $\{|\,\rangle, Q_{\alpha-}|\,\rangle, Q_{\alpha-}Q_{\beta-}|\,\rangle\}$ will then decompose as $[(p,q)\times\{t\}+(p,q+\frac{1}{2})\times\{t-1\}+(p,q)-\frac{1}{2})\times\{t-1\}+(p,q)\times\{t-2\}]$ under $\mathrm{O}(3,1)\times\mathrm{U}(1)$. We find that this does in fact constitute an irreducible representation of $\mathscr{H}$.

The generators of $\mathscr{G}$ can be realised as differential operators in the coset space $\mathscr{G}/\mathscr{H}$ and as matrix representatives in the representation space of $\mathscr{H}_0$. Their explicit form is

$$\begin{aligned}
P_\mu &= -\mathrm{i}\partial_\mu \\
J_{\mu\nu} &= \mathrm{i}(\eta_{\nu\rho}x^\rho\partial_\mu-\eta_{\mu\rho}x^\rho\partial_\nu)+\mathrm{i}\bar{\theta}^{\beta a}(\bar{\sigma}_{\mu\nu})_\beta{}^{\dot{\alpha}}\partial_{\dot{\alpha}a}-J^0{}_{\mu\nu} \\
\bar{Q}_{\dot{\alpha}\pm} &= -\mathrm{i}\partial_{\dot{\alpha}\pm}\pm 2\bar{\theta}^{\beta\mp}\varepsilon_{\dot{\alpha}\beta}\bar{Z}^0 \\
Q_{\alpha\pm} &= 4\bar{\theta}^{\dot{\alpha}\mp}(\sigma^\mu)_{\alpha\dot{\alpha}}\partial_\mu - Q^0{}_{\alpha\pm} \\
Z &= -Z^0 \\
T &= \bar{\theta}^{\dot{\alpha}+}\partial_{\dot{\alpha}+}-\bar{\theta}^{\dot{\alpha}-\partial}{}_{\dot{\alpha}-}-T^0
\end{aligned} \tag{5}$$

where $\partial_{\dot{\alpha}a}=\partial/\partial\bar{\theta}^{\dot{\alpha}a}$, $\partial_\mu=\partial/\partial x^\mu$ and $J^0{}_{\mu\nu}$, $Z^0$, $T^0$ and $Q^0{}_{\alpha a}$ are the matrix representations of the 'little superalgebra' which determines the external transformation rules of the superfield. Suitable forms for $Z^0$ and $T^0$ are

$$Z^0=-Z\begin{bmatrix}\delta_a{}^c & & 0 & \\ & \Pi^{+\beta c}_{\alpha a} & & \\ 0 & & \Pi^{-\beta c}_{\alpha a} & \\ & & & \delta_b{}^d\end{bmatrix} \tag{6}$$

$$T^0=\begin{bmatrix}-T\delta_a{}^c & & 0 & \\ & -(T-1)\Pi^{+\beta c}_{\alpha a} & & \\ 0 & & -(T-1)\Pi^{-\beta c}_{\alpha a} & \\ & & & -(T-2)\delta_b{}^d\end{bmatrix}. \tag{7}$$

Spin $p\pm\frac{1}{2}$ and spin $q\pm\frac{1}{2}$ projectors are denoted by $\Pi^{\pm\dot{\beta}}_{\dot{\alpha}}$ and $\Pi^{\pm\beta}_{\alpha}$ respectively (see appendix 1 for details). $J^0_{\mu\nu}$ may be expressed in terms of these projectors; however, an explicit form is not required for the following analysis. The algebra satisfied by $\mathscr{G}$ requires matrices $Q^0{}_{\alpha a}$ which satisfy $\{Q^0{}_{\alpha a}, Q^0{}_{\beta b}\}=4\mathrm{i}\varepsilon_{\alpha\beta}\varepsilon_{ab}Z^0$. These are found to be of the form

$$Q^0{}_{\gamma+}=+y\mu\begin{bmatrix}0 & 0 & 0 & 0\\ (\Pi^+\varepsilon)_{\alpha\gamma a}{}^c & 0 & 0 & 0\\ (\Pi^-\varepsilon)_{\alpha\gamma a}{}^c & 0 & 0 & 0\\ 0 & \Pi^{+\beta c}_{\gamma b} & \Pi^{-\beta c}_{\gamma b} & 0\end{bmatrix} \tag{8}$$

$$Q^0{}_{\gamma-}=+y\mu\begin{bmatrix}0 & -\Pi^{+\beta c}_{\gamma a} & -\Pi^{-\beta c}_{\gamma a} & 0\\ 0 & 0 & 0 & (\Pi^+\varepsilon)_{\alpha\gamma a}{}^d\\ 0 & 0 & 0 & (\Pi^-\varepsilon)_{\alpha\gamma a}{}^d\\ 0 & 0 & 0 & 0\end{bmatrix} \tag{9}$$

where $\mu=\sqrt{Z}$ and $y=(1+\mathrm{i})\sqrt{2}$ has been chosen simply to render the most symmetrical form for $Q^0_{\gamma\pm}$. The only essential requirement for these coefficients is that their product is $4\mathrm{i}Z$.

The superfields $\Phi_A(x^\mu, \bar\theta^{\dot\alpha a})$ form a representation of $\mathscr{H}$ labelled by $\{(p, q), T, Z\}$:

$$\Phi_A(x^\mu, \bar\theta^{\dot\beta a}) = \begin{pmatrix} V_a(x^\mu, \bar\theta^{\dot\beta a}) \\ \Sigma^+_{a\alpha}(x^\mu, \bar\theta^{\dot\alpha a}) \\ \Sigma^-_{a\alpha}(x^\mu, \bar\theta^{\dot\alpha a}) \\ W_a(x^\mu, \bar\theta^{\dot\alpha a}) \end{pmatrix}. \tag{10}$$

The general form of the superfield when expanded in $\bar\theta^{\dot\alpha a}$ is (spin-$q$ indices will be suppressed in the following work; $\bar\theta$ monomials are defined in appendix 2 together with some useful identities):

$$\Phi_A(x^\mu, \bar\theta^{\dot\alpha a}) = \begin{pmatrix} A \\ \alpha^+_\alpha \\ \alpha^-_\alpha \\ a \end{pmatrix} + \bar\theta^{\dot\alpha a}\Sigma_m \begin{pmatrix} \Psi^m_{\dot\alpha a} \\ P^{m+}_{\dot\alpha\alpha a} \\ P^{m-}_{\dot\alpha\alpha a} \\ \psi^m_{\dot\alpha a} \end{pmatrix} + (\bar\theta\bar\theta)^{ab}\Sigma_k \begin{pmatrix} F_k \\ \phi^+_{\alpha k} \\ \phi^-_{\alpha k} \\ f_k \end{pmatrix}^{\dagger}$$

$$+ (\bar\theta\bar\theta)^{\dot\alpha\dot\beta}\Sigma_l \begin{pmatrix} G^l_{\dot\alpha\dot\beta} \\ \gamma^{l+}_{\dot\alpha\dot\beta\alpha} \\ \gamma^{l-}_{\dot\alpha\dot\beta\alpha} \\ g^l_{\dot\alpha\dot\beta} \end{pmatrix} + (\bar\theta^3)^{\dot\alpha a}\Sigma_m \begin{pmatrix} \Omega^m_{\dot\alpha a} \\ W^{m+}_{\dot\alpha\alpha a} \\ W^{m-}_{\dot\alpha\alpha a} \\ \omega^m_{\dot\alpha a} \end{pmatrix} + (\bar\theta^4) \begin{pmatrix} D \\ \delta^+_\alpha \\ \delta^-_\alpha \\ d \end{pmatrix} \tag{11}$$

where $m = +, -$ refer to spin $p+\frac{1}{2}$ and $p-\frac{1}{2}$ projections, $l = 0, +, -$ refer to spin $p$, $p+1$ and $p-1$ projections, $a = +, -$ refer to $T+1$ and $T-1$ projections and $k = 0, +, -$ refer to $T$, $T+2$, $T-2$ projections. All component fields are functions of $x^\mu$.

To determine the irreducibility or indecomposability of $\Phi_A(x^\mu, \bar\theta^{\dot\alpha a})$ it is necessary to introduce appropriate field redefinitions for the component fields and examine their variations under the odd generators. To aid in this we recall that the algebra realised by (5) may be extended to include the generators $\bar S_{\dot\alpha a} = \partial_{\dot\alpha a}$ yielding the extended algebra $\bar{\mathscr{G}}$. As we have noted, since the general superfield $\Phi_A(x^\mu, \bar\theta^{\dot\alpha a})$ is still a representation of $\bar{\mathscr{G}}$ it is expected to be reducible under $\mathscr{G}$. To find the irreducible representations of $\mathscr{G}$ contained in $\Phi$ we proceed as follows. Given an irreducible representation of $\mathscr{G}$ with highest weight vector $\Lambda = |(p, q), T, Z\rangle$, such that

$$Q_{\alpha+}\Lambda = \bar Q_{\dot\alpha+}\Lambda = \bar S_{\dot\alpha+}\Lambda = 0$$

a basis from $\bar{\mathscr{G}}$ may be obtained from the four vectors $\Lambda$, $\Pi^{+}{}_{\dot\alpha}{}^{\beta}\bar S_{\dot\alpha-}\Lambda$, $\Pi^{-}{}_{\beta}{}^{\alpha}\bar S_{\dot\beta-}\Lambda$, $(\bar S_{\dot\alpha-})^2\Lambda$ by acting with monomials of $\bar Q_{\dot\alpha-}$ and $Q_{\alpha-}$. This suggests that a superfield $\Phi_A(x^\mu, \bar\theta^{\dot\alpha a})$, which carries a representation $((p, q), T, Z)$ of $\mathscr{H}$, contains four irreducible representations of $\mathscr{G}$. This is indeed found to be the case, with $\Lambda = |(p, q), T+2, Z\rangle$ and consequently $\Pi^{+}{}_{\dot\alpha}{}^{\beta}\bar S_{\dot\beta-}\Lambda = |(p+\frac{1}{2}, q), T+1, Z\rangle$, $\Pi^{-}{}_{\dot\alpha}{}^{\beta}\bar S_{\dot\beta-}\Lambda = |(p-\frac{1}{2}, q), T+1, Z\rangle$ and $(\bar S_{\dot\alpha-})^2\Lambda = |(p, q), T, Z\rangle$. Each of these multiplets contains sixteen fields with weights (relative to the central values) as shown in table 1.

To obtain the basis which renders the irreducible multiplets of $\mathscr{G}$ evident we proceed as follows. From the superfield it is apparent that the highest weight vector, $\Lambda$, is $F_+$ since $\delta_{Q_{\gamma+}}F_+ = \delta_{\bar Q_{\dot\gamma+}}F_+ = \delta_{\bar S_{\dot\gamma+}}F_+ = 0$. The variations of $F_+$ under $Q_{\gamma-}$ and $\bar Q_{\dot\gamma-}$ are:

$$\delta_{Q_\gamma^-}F_+ = -y\mu(\phi^+_{\gamma+} + \gamma^-_{\gamma+}) + 2(\sigma^\mu)_\gamma{}^{\dot\alpha}(\partial_\mu\Psi^+_{\dot\alpha+} + \partial_\mu\Psi^-_{\dot\alpha+}) \tag{12}$$

† To be precise this term should read, considering for example the top component only,

$$(\bar\theta\bar\theta)^{ab}F_{ab} = (\bar\theta\bar\theta)^{++}F_{++} + (\bar\theta\bar\theta)^{--}F_{--} + (\bar\theta\bar\theta)^{+-}F_{+-} + (\bar\theta\bar\theta)^{-+}F_{-+}.$$

Thus we define $F_+ = F_{++}$, $F_- = F_{--}$ and $F_0 = F_{+-} + F_{-+}$.

**Table 1.** Weights and defining fields of the four irreducible multiplets of $\mathscr{G}$ contained in the superfield. The fields of the $\Lambda_1$, $\Lambda_3$ and $\Lambda_4$ multiplets are defined as proportional to $\Pi Q_{\alpha-}\Pi\bar{Q}_{\dot\alpha-}$ acting on the highest weight vectors $\tilde{F}_+$, $\tilde{\Psi}^+{}_{\dot\alpha+}$ and $\tilde{\Psi}^-{}_{\dot\alpha-}$ respectively. The fields of the $\Lambda_2$ multiplet are defined as proportional to $\Pi Q_{\alpha+}\Pi\bar{Q}_{\dot\alpha+}$ acting on the lowest weight vector $\tilde{f}_-$.

| | $p$, | $q$, | $T$ | $\Lambda_1$ | $p$, | $q$, | $T$ | $\Lambda_2$ | $p$, | $q$, | $T$ | $\Lambda_3$ | $p$, | $q$, | $T$ | $\Lambda_4$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\Lambda_i$ | 0, | 0, | +2 | $\tilde{F}_+$ | 0, | 0, | −4 | $\tilde{f}_-$ | $\frac{1}{2}$, | 0, | +1 | $\tilde{\Psi}^+{}_{\dot\alpha+}$ | $-\frac{1}{2}$, | 0, | +1 | $\tilde{\Psi}^-{}_{\dot\alpha+}$ |
| $Q_{\alpha\pm}\Lambda_i$ | 0, | $\frac{1}{2}$, | +1 | $\tilde{\phi}^+{}_{\alpha+}$ | 0, | $\frac{1}{2}$, | −3 | $\tilde{\phi}^+{}_{\alpha-}$ | $\frac{1}{2}$, | $\frac{1}{2}$, | 0 | $\tilde{P}^{++}_{\dot\alpha\alpha+}$ | $-\frac{1}{2}$, | $\frac{1}{2}$, | 0 | $\tilde{P}^{-+}_{\dot\alpha\alpha+}$ |
| | 0, | $-\frac{1}{2}$, | +1 | $\tilde{\phi}^-{}_{\alpha+}$ | 0, | $-\frac{1}{2}$, | −3 | $\tilde{\phi}^-{}_{\alpha-}$ | $\frac{1}{2}$, | $-\frac{1}{2}$, | 0 | $\tilde{P}^{+-}_{\dot\alpha\alpha+}$ | $-\frac{1}{2}$, | $-\frac{1}{2}$, | 0 | $\tilde{P}^{--}_{\dot\alpha\alpha+}$ |
| $\bar{Q}_{\dot\alpha\pm}\Lambda_i$ | $\frac{1}{2}$, | 0, | +1 | $\tilde{\Omega}^+{}_{\dot\alpha+}$ | $\frac{1}{2}$, | 0, | −3 | $\tilde{\omega}^+{}_{\dot\alpha-}$ | 1, | 0, | 0 | $\tilde{G}^+{}_{\dot\alpha\beta}$ | −1, | 0, | 0 | $\tilde{G}^-_{\dot\alpha\beta}$ |
| | $-\frac{1}{2}$, | 0, | +1 | $\tilde{\Omega}^-{}_{\dot\alpha+}$ | $-\frac{1}{2}$, | 0, | −3 | $\tilde{\omega}^-{}_{\dot\alpha-}$ | 0, | 0, | 0 | $\tilde{F}_0$ | 0, | 0, | 0 | $\tilde{G}^0_{\dot\alpha\beta}$ |
| $Q_{\alpha\pm}Q_{\beta\pm}\Lambda_i$ | 0, | 0, | 0 | $\tilde{f}_+$ | 0, | 0, | −2 | $\tilde{F}_-$ | $\frac{1}{2}$, | 0, | −1 | $\tilde{\psi}^+{}_{\dot\alpha+}$ | $-\frac{1}{2}$, | 0, | −1 | $\tilde{\psi}^-{}_{\dot\alpha+}$ |
| $\bar{Q}_{\dot\alpha\pm}\bar{Q}_{\beta\pm}\Lambda_i$ | 0, | 0, | 0 | $\tilde{D}$ | 0, | 0, | −2 | $\tilde{a}$ | $\frac{1}{2}$, | 0, | −1 | $\tilde{\Psi}^+{}_{\dot\alpha-}$ | $-\frac{1}{2}$, | 0, | −1 | $\tilde{\Psi}^-{}_{\dot\alpha-}$ |
| $Q_{\alpha\pm}\bar{Q}_{\dot\alpha\pm}\Lambda_i$ | $\frac{1}{2}$, | $\frac{1}{2}$, | 0 | $\tilde{W}^{++}_{\dot\alpha\alpha+}$ | $\frac{1}{2}$, | $\frac{1}{2}$, | −2 | $\tilde{W}^{++}_{\dot\alpha\alpha-}$ | 1, | $\frac{1}{2}$, | −1 | $\tilde{\gamma}^{++}_{\dot\alpha\beta\alpha}$ | −1, | $\frac{1}{2}$, | −1 | $\tilde{\gamma}^{-+}_{\dot\alpha\beta\alpha}$ |
| | $\frac{1}{2}$, | $-\frac{1}{2}$, | 0 | $\tilde{W}^{+-}_{\dot\alpha\alpha+}$ | $\frac{1}{2}$, | $-\frac{1}{2}$, | −2 | $\tilde{W}^{+-}_{\dot\alpha\alpha-}$ | 1, | $-\frac{1}{2}$, | −1 | $\tilde{\gamma}^{+-}_{\dot\alpha\beta\alpha}$ | −1, | $-\frac{1}{2}$, | −1 | $\tilde{\gamma}^{--}_{\dot\alpha\beta\alpha}$ |
| | $-\frac{1}{2}$, | $\frac{1}{2}$, | 0 | $\tilde{W}^{-+}_{\dot\alpha\alpha+}$ | $-\frac{1}{2}$, | $\frac{1}{2}$, | −2 | $\tilde{W}^{-+}_{\dot\alpha\alpha-}$ | 0, | $\frac{1}{2}$, | −1 | $\tilde{\phi}^+{}_{\alpha0}$ | 0, | $\frac{1}{2}$, | −1 | $\tilde{\gamma}^{0+}_{\dot\alpha\beta\alpha}$ |
| | $-\frac{1}{2}$, | $-\frac{1}{2}$, | 0 | $\tilde{W}^{--}_{\dot\alpha\alpha+}$ | $-\frac{1}{2}$, | $-\frac{1}{2}$, | −2 | $\tilde{W}^{--}_{\dot\alpha\alpha-}$ | 0, | $-\frac{1}{2}$, | −1 | $\tilde{\phi}^-{}_{\alpha0}$ | 0, | $-\frac{1}{2}$, | −1 | $\tilde{\gamma}^{0-}_{\dot\alpha\beta\alpha}$ |
| $Q_{\alpha\pm}\bar{Q}_{\dot\alpha\pm}\bar{Q}_{\beta\pm}\Lambda_i$ | 0, | $\frac{1}{2}$, | −1 | $\tilde{\delta}^+{}_\alpha$ | 0, | $\frac{1}{2}$, | −1 | $\tilde{\alpha}^+{}_\alpha$ | $\frac{1}{2}$, | $\frac{1}{2}$, | −2 | $\tilde{P}^{++}_{\frac{1}{2}\alpha-}$ | $-\frac{1}{2}$, | $\frac{1}{2}$, | −2 | $\tilde{P}^{-+}_{\frac{1}{2}\alpha-}$ |
| | 0, | $-\frac{1}{2}$, | −1 | $\tilde{\delta}^-{}_\alpha$ | 0, | $-\frac{1}{2}$, | −1 | $\tilde{\alpha}^-{}_\alpha$ | $\frac{1}{2}$, | $-\frac{1}{2}$, | −2 | $\tilde{P}^{+-}_{\dot\alpha\alpha-}$ | $-\frac{1}{2}$, | $-\frac{1}{2}$, | −2 | $\tilde{P}^{--}_{\dot\alpha\alpha-}$ |
| $\bar{Q}_{\dot\alpha\pm}Q_{\alpha\pm}Q_{\beta\pm}\Lambda_i$ | $\frac{1}{2}$, | 0, | −1 | $\tilde{\omega}^+_{\dot\alpha+}$ | $\frac{1}{2}$, | 0, | −1 | $\tilde{\Omega}^+{}_{\dot\alpha-}$ | 1, | 0, | −2 | $\tilde{g}^+{}_{\dot\alpha\beta}$ | −1, | 0, | −2 | $\tilde{g}^-{}_{\dot\alpha\beta}$ |
| | $-\frac{1}{2}$, | 0, | −1 | $\tilde{\omega}^-_{\dot\alpha+}$ | $-\frac{1}{2}$, | 0, | −1 | $\tilde{\Omega}^-{}_{\dot\alpha-}$ | 0, | 0, | −2 | $\tilde{f}_0$ | 0, | 0, | −2 | $\tilde{g}^0{}_{\dot\alpha\beta}$ |
| $Q_{\alpha\pm}Q_{\beta\pm}\bar{Q}_{\dot\alpha\pm}\bar{Q}_{\beta\pm}\Lambda_i$ | 0, | 0, | −2 | $\tilde{d}$ | 0, | 0, | 0 | $\tilde{A}$ | $\frac{1}{2}$, | 0, | −3 | $\tilde{\psi}^+{}_{\dot\alpha-}$ | $-\frac{1}{2}$, | 0, | −3 | $\tilde{\psi}^-{}_{\dot\alpha-}$ |

$$\delta_{\bar{Q}_{\gamma-}}F_+ = \tfrac{3}{2}\mathrm{i}(\Omega^+_{\dot\gamma+}+\Omega^-_{\dot\gamma+}) - \bar{\mu}^2(\Psi^+_{\dot\gamma+}+\Psi^-_{\dot\gamma+}). \tag{13}$$

From (12) and (13) we project spin $q\pm\frac{1}{2}$ and spin $p\pm\frac{1}{2}$ states respectively, and define new fields proportional to those states. Thus, explictly, we have

$$\Pi^{\pm}{}_{\alpha}{}^{\gamma}\delta_{Q_{\gamma-}}F_+ = -y\mu\tilde{\phi}^{\pm}_{\alpha+} \tag{14}$$

$$\Pi^{\pm}{}_{\dot\alpha}{}^{\dot\gamma}\delta_{Q_{\gamma-}}F_+ = \tilde{\Omega}^{\pm}{}_{\dot\alpha+} \tag{15}$$

where

$$\tilde{\phi}^{\pm}{}_{\alpha+} = \phi^{\pm}{}_{\alpha+} - (2/y\mu)(\sigma_{\pm}{}^{\mu})_{\alpha}{}^{\beta}(\partial_\mu\Psi^+_{\dot\alpha+}+\partial_\mu\Psi^-_{\dot\alpha+})$$

and

$$\tilde{\Omega}^{\pm}{}_{\dot\alpha+} = \tfrac{3}{2}\mathrm{i}\Omega^{\pm}{}_{\dot\alpha+} - \bar{\mu}^2\Psi^{\pm}{}_{\dot\alpha+}$$

(see table 2 for notation).

We now consider the variation of each of these ~ fields under $Q_{\gamma-}$ and $\bar{Q}_{\dot\gamma-}$ and define new ~ fields by projecting Lorentz eigenstates from the field variations as in (14) and (15). This procedure is simply repeated until a basis for the sixteen states of this multiplet has been generated. This basis is given in table 2(*a*).

For the multiplet characterised by $(\bar{S}_{\dot\alpha-})^2\Lambda$ the judicious choice is to consider a lowest weight vector $\bar{\Lambda} = |(p,q), T-4, Z\rangle$ such that $Q_{\alpha-}\bar{\Lambda} = \bar{Q}_{\dot\alpha-}\bar{\Lambda} = \bar{S}_{\dot\alpha-}\bar{\Lambda} = 0$ and obtain the remaining states of the multiplet by acting with monomials of $Q_{\alpha+}$ and $\bar{Q}_{\dot{\alpha}+}$ on $\bar{\Lambda}$. From the superfield we find that $f_-$ is the field corresponding to $\bar{\Lambda}$, since $\delta_{Q\alpha-}f_- = \delta_{\bar{Q}_{\dot{\alpha}}}f_- = \delta_{\bar{S}_{\dot{\alpha}}}f_- = 0$. By analogy with the $F_+$ multiplet we now determine the variations of $f_-$ under $Q_{\alpha+}$ and $\bar{Q}_{\dot{\alpha}+}$ and define new fields as proportional to the Lorentz eigenstates projected from these variations. Again by repeated application of this procedure we obtain a basis for the sixteen states of this multiplet. This basis is given in table 2$(b)$.

For the remaining two multiplets characterised by $\Pi^{\pm}{}_{\dot{\alpha}}{}^{\dot{\beta}}\bar{S}_{\dot{\beta}-}\Lambda$, we see from the superfield that the highest weight states will be some linear combination of $\Omega^{\pm}{}_{\dot{\alpha}+}$ and $\Psi^{\pm}{}_{\dot{\alpha}+}$ which is linearly independent of $\tilde{\Omega}^{\pm}{}_{\dot{\alpha}+}$. For simplicity we choose $\tilde{\Psi}^{\pm}{}_{\dot{\alpha}+} = \Psi^{\pm}{}_{\dot{\alpha}+}$ for which $\delta_{Q_{\gamma+}}\tilde{\Psi}^{\pm}{}_{\dot{\alpha}+} = 0$ and $\delta_{\bar{Q}_{\dot{\gamma}+}}\tilde{\Psi}^{\pm}_{\dot{\alpha}+} = \delta_{\bar{S}_{\dot{\gamma}+}}\tilde{\Psi}^{\pm}{}_{\dot{\alpha}+} = 2\mathrm{i}(\Pi^{\pm}\varepsilon)_{\dot{\alpha}\dot{\lambda}}F_+$. Thus as will be seen presently, $\tilde{\Psi}^{\pm}{}_{\dot{\alpha}+}$, are highest weight vectors, modulo coset elements, $F_+$. Again by analogy with the $F_+$ multiplet, the bases for the $\Psi^{\pm}{}_{\dot{\alpha}+}$ multiplets are obtained by acting

**Table 2.** Basis for $(a)$ the $\Lambda$, $(b)$ the $(\bar{S}_{\dot{\alpha}-})^2\Lambda$, $(c)$ the $\Pi^{+}{}_{\dot{\alpha}}{}^{\beta}\bar{S}_{\dot{\beta}-}\Lambda$ and $(d)$ the $\Pi^{-}{}_{\dot{\alpha}}{}^{\dot{\beta}}\bar{S}_{\dot{\beta}}$ multiplets. In these tables the following notation has been adopted:

$$(\sigma^{\mu}_{\pm})_{\dot{\alpha}\alpha} = \Pi^{\pm}{}_{\alpha}{}^{\beta}(\sigma^{\mu})_{\dot{\alpha}\beta} \qquad (\sigma^{\mu})^{\pm}{}_{\dot{\alpha}\alpha} = \Pi^{\pm}{}_{\dot{\alpha}}{}^{\dot{\beta}}(\sigma^{\mu})_{\dot{\beta}\alpha}$$

$$M^{\pm}_{\pm\dot{\alpha}\dot{\beta}\dot{\delta}}{}^{\dot{\gamma}} = \frac{1}{p^{\pm}}\hat{M}_{\dot{\alpha}\dot{\beta}}\delta_{\dot{\lambda}}{}^{\dot{\gamma}} + \varepsilon_{\dot{\lambda}\dot{\alpha}}\delta_{\dot{\beta}}{}^{\dot{\gamma}} + \varepsilon_{\dot{\lambda}\dot{\beta}}\delta_{\dot{\alpha}}{}^{\dot{\gamma}}$$

where $p^{+} = p$ and $p^{-} = -p-1$, and

$$G^{0}_{\pm\dot{\alpha}\beta} = \Pi^{\pm}{}_{\dot{\alpha}}{}^{\dot{\gamma}}G^{0}_{\dot{\gamma}\beta} \qquad \gamma^{0\pm}_{\pm\dot{\alpha}\beta\alpha} \qquad g^{0}_{\pm\dot{\alpha}\beta}$$

are similarly defined. See appendix 1 for further discusion of the properties of the spin $p\times\frac{1}{2}$ projectors, $\Pi^{\pm}{}_{\dot{\alpha}}{}^{\beta}$, and the spin $q\times\frac{1}{2}$ projectors, $\Pi^{\pm}{}_{\alpha}{}^{\beta}$.

$(a)$

$$\tilde{F}_+ = F_+$$

$$\tilde{\phi}^{\pm}{}_{\alpha+} = \phi^{\pm}{}_{\alpha+} - \frac{2}{y\mu}(\sigma^{\mu}_{\pm})_{\alpha}{}^{\beta}[\partial_{\mu}\Psi^{+}{}_{\beta+} + \partial_{\mu}\Psi^{-}{}_{\beta+}]$$

$$\tilde{\Omega}^{\pm}{}_{\dot{\alpha}+} = \tfrac{3}{2}\mathrm{i}\Omega^{\pm}{}_{\dot{\alpha}+} - \bar{\mu}^{2}\Psi^{\pm}{}_{\dot{\alpha}+}$$

$$\tilde{f}_+ = f_+ + \frac{8}{y^2\mu^2}\partial^2 A - \frac{2}{y\mu}(\sigma^{\mu})^{\dot{\alpha}\alpha}[\partial_{\mu}P^{++}_{\dot{\alpha}\alpha+} + \partial_{\mu}P^{+-}_{\dot{\alpha}\alpha+} + \partial_{\mu}P^{-+}_{\dot{\alpha}\alpha+} + \partial_{\mu}P^{--}_{\dot{\alpha}\alpha+}]$$

$$\tilde{D} = 3D + \mathrm{i}\bar{\mu}^2F_0 - \bar{\mu}^4 A$$

$$\tilde{W}^{\pm\pm}_{\dot{\alpha}\alpha+} = \tfrac{3}{2}\mathrm{i}\,W^{\pm\pm}_{\dot{\alpha}\alpha+} - \bar{\mu}^2P^{\pm\pm}_{\dot{\alpha}\alpha+} + \frac{2\mathrm{i}}{y\mu}(\sigma^{\mu}_{\pm})^{\pm}{}_{\dot{\alpha}\alpha}\partial_{\mu}F_0 - \frac{4\bar{\mu}^2}{y\mu}(\sigma^{\mu}_{\pm})^{\pm}{}_{\dot{\alpha}\alpha}\partial_{\mu}A - \frac{4\mathrm{i}}{y\mu}(\sigma^{\mu}_{\pm})_{\alpha}{}^{\beta}[\partial_{\mu}G^{\pm}{}_{\beta\dot{\alpha}} + \partial_{\mu}G^{0}{}_{\pm\beta\dot{\alpha}}]$$

$$\tilde{\delta}^{\pm}{}_{\alpha} = 3\delta^{\pm}{}_{\alpha} + \mathrm{i}\bar{\mu}^2\phi^{\pm}{}_{\alpha 0} - \bar{\mu}^4\alpha^{\pm}{}_{\alpha} - \frac{2\mathrm{i}\bar{\mu}^2}{y\mu}(\sigma^{\mu}_{\pm})_{\alpha}{}^{\dot{\alpha}}[\partial_{\mu}\Psi^{+}{}_{\dot{\alpha}-} + \partial_{\mu}\Psi^{-}{}_{\dot{\alpha}-}] + \frac{3}{y\mu}(\sigma^{\mu}_{\pm})_{\alpha}{}^{\dot{\alpha}}[\partial_{\mu}\Omega^{+}{}_{\dot{\alpha}-} + \partial_{\mu}\Omega^{-}{}_{\dot{\beta}}]$$

$$\tilde{\omega}^{\pm}{}_{\dot{\alpha}+} = \tfrac{3}{2}\mathrm{i}\omega^{\pm}{}_{\dot{\alpha}+} - \bar{\mu}^2\psi^{\pm}{}_{\dot{\alpha}+} - \frac{8\mathrm{i}}{y^2\mu^2}\partial^2\Psi^{\pm}{}_{\dot{\alpha}-} + \frac{2\mathrm{i}}{y\mu}(\sigma^{\mu})^{\pm}{}_{\dot{\alpha}}{}^{\alpha}[\partial_{\mu}\phi^{+}{}_{\alpha 0} + \partial_{\mu}\phi^{-}{}_{\alpha 0}] - \frac{4\bar{\mu}^2}{y\mu}(\sigma^{\mu})^{\pm}{}_{\dot{\alpha}}{}^{\alpha}[\partial_{\mu}\alpha^{+}{}_{\alpha} + \partial_{\mu}\alpha^{-}{}_{\alpha}]$$
$$- \frac{4\mathrm{i}}{y\mu}(\sigma^{\mu})^{\dot{\beta}\alpha}[\partial_{\mu}\gamma^{\pm\pm}_{\dot{\beta}\dot{\alpha}\alpha} + \partial_{\mu}\gamma^{\pm-}_{\dot{\beta}\dot{\alpha}\alpha} + \partial_{\mu}\gamma^{0+}_{\pm\dot{\beta}\dot{\alpha}\alpha} + \partial_{\mu}\gamma^{0-}_{\pm\dot{\beta}\alpha\alpha}]$$

$$\tilde{d} = 3d + \mathrm{i}\bar{\mu}^2 f_0 - \bar{\mu}^4 a - \frac{8}{y^2\mu^2}\partial^2F_- - \frac{2\mathrm{i}\bar{\mu}^2}{y\mu}(\sigma^{\mu})^{\dot{\alpha}\alpha}[\partial_{\mu}P^{++}_{\dot{\alpha}\alpha-} + \partial_{\mu}P^{+-}_{\dot{\alpha}\alpha-} + \partial_{\mu}P^{-+}_{\dot{\alpha}\alpha-} + \partial_{\mu}P^{--}_{\dot{\alpha}\alpha-}]$$
$$+ \frac{3}{y\mu}(\sigma^{\mu})^{\dot{\alpha}\alpha}[\partial_{\mu}W^{++}_{\dot{\alpha}\alpha-} + \partial_{\mu}W^{+-}_{\dot{\alpha}\alpha-} + \partial_{\mu}W^{-+}_{\dot{\alpha}\alpha-} + \partial_{\mu}W^{--}_{\dot{\alpha}\alpha-}]$$

**Table 2.** (continued)

(*b*)

$$\tilde{f}_- = f_-$$

$$\tilde{\phi}^{\pm}{}_{\alpha-} = \phi^{\pm}{}_{\alpha-} + \frac{2}{y\mu}(\sigma^{\mu}_{\pm})_{\alpha}{}^{\dot\alpha}[\partial_\mu \psi^{+}{}_{\dot\alpha-} + \partial_\mu \psi^{-}{}_{\dot\alpha}]$$

$$\tilde{\omega}^{\pm}{}_{\dot\alpha-} = \tfrac{3}{2}\mathrm{i}\omega^{\pm}{}_{\dot\alpha-} - \bar\mu^2\psi^{\pm}{}_{\dot\alpha-}$$

$$\tilde{F}_- = F_- - \frac{8}{y^2\mu^2}\partial^2 a - \frac{2}{y\mu}(\sigma^\mu)^{\dot\alpha\alpha}[\partial_\mu P^{++}_{\dot\alpha\alpha-} + \partial_\mu P^{+-}_{\dot\alpha\alpha-} + \partial_\mu P^{-+}_{\dot\alpha\alpha-} + \partial_\mu P^{--}_{\alpha\alpha-}]$$

$$\tilde{a} = 3d - \mathrm{i}\bar\mu^2 f_0 - \bar\mu^4 a$$

$$\tilde{W}^{\pm\pm}_{\dot\alpha\alpha-} = \tfrac{3}{2}\mathrm{i}\, W^{\pm\pm}_{\dot\alpha\alpha-} - \bar\mu^2 P^{\pm\pm}_{\dot\alpha\alpha-} + \frac{2\mathrm{i}}{y\mu}(\sigma^\mu_{\pm})^{\pm}{}_{\dot\alpha\alpha}\partial_\mu f_0 + \frac{4\bar\mu^2}{y\mu}(\sigma^\mu_{\pm})^{\pm}{}_{\dot\alpha\alpha}\partial_\mu a + \frac{4\mathrm{i}}{y\mu}(\sigma^\mu_{\pm})_{\alpha}{}^{\beta}[\partial_\mu g^{\pm}{}_{\beta\dot\alpha} + \partial_\mu g^{0}{}_{\pm\beta\dot\alpha}]$$

$$\tilde{\alpha}^{\pm}_{\alpha} = 3\delta^{\pm}{}_{\alpha} - \mathrm{i}\bar\mu^2\phi^{\pm}{}_{\alpha 0} - \bar\mu^4\alpha^{\pm}{}_{\alpha} - \frac{2\mathrm{i}\bar\mu^2}{y\mu}(\sigma^\mu_{\pm})_{\alpha}{}^{\dot\alpha}[\partial_\mu\psi^{+}{}_{\dot\alpha+} + \partial_\mu\psi^{-}{}_{\dot\alpha+}] + \frac{3}{y\mu}(\sigma^\mu_{\pm})_{\alpha}{}^{\dot\alpha}[\partial_\mu\omega^{+}{}_{\dot\alpha+} + \partial_\mu\omega^{-}{}_{\dot\alpha+}]$$

$$\tilde{\Omega}^{\pm}{}_{\dot\alpha-} = \tfrac{3}{2}\mathrm{i}\Omega^{\pm}{}_{\dot\alpha-} - \bar\mu^2\Psi^{\pm}{}_{\dot\alpha-} - \frac{8\mathrm{i}}{y^2\mu^2}\partial^2\psi^{\pm}{}_{\dot\alpha+} - \frac{2\mathrm{i}}{y\mu}(\sigma^\mu)^{\pm}{}_{\dot\alpha}{}^{\alpha}[\partial_\mu\phi^{+}{}_{\alpha 0} + \partial_\mu\phi^{-}{}_{\alpha 0}] - \frac{4\bar\mu^2}{y\mu}(\sigma^\mu)^{\pm}{}_{\dot\alpha}{}^{\alpha}[\partial_\mu\alpha^{+}{}_{\alpha} + \partial_\mu\alpha^{-}{}_{\alpha}]$$
$$- \frac{4\mathrm{i}}{y\mu}(\sigma^\mu)^{\beta\alpha}[\partial_\mu\gamma^{\pm+}_{\beta\dot\alpha\alpha} + \partial_\mu\gamma^{\pm-}_{\dot\beta\beta\alpha} + \partial_\mu\gamma^{0+}_{\pm\beta\dot\alpha\alpha} + \partial_\mu\gamma^{0-}_{\pm\beta\dot\alpha\alpha}]$$

$$\tilde{A} = 3D - \mathrm{i}\bar\mu^2 F_0 - \bar\mu^4 A + \frac{8}{y^2\mu^2}\partial^2 f_+ - \frac{2\mathrm{i}\bar\mu^2}{y\mu}(\sigma^\mu)^{\dot\alpha\alpha}[\partial_\mu P^{++}_{\dot\alpha\alpha} + \partial_\mu P^{+-}_{\dot\alpha\alpha+} + \partial_\mu P^{-+}_{\dot\alpha\alpha+} + \partial_\mu P^{--}_{\dot\alpha\alpha+}]$$
$$+ \frac{3}{y\mu}(\sigma^\mu)^{\dot\alpha\alpha}[\partial_\mu W^{++}_{\dot\alpha\alpha+} + \partial_\mu W^{+-}_{\dot\alpha\alpha+} + \partial_\mu W^{-+}_{\dot\alpha\alpha+} + \partial_\mu W^{--}_{\dot\alpha\alpha+}]$$

(*c*)

$$\tilde{\Psi}^{+}{}_{\dot\alpha+} = \Psi^{+}{}_{\dot\alpha+}$$

$$\tilde{P}^{+\pm}_{\dot\alpha\alpha+} = P^{+\pm}_{\dot\alpha\alpha+} + \frac{4}{y\mu}(\sigma^\mu_{\pm})^{+}{}_{\dot\alpha\alpha}\partial_\mu A$$

$$\tilde{G}^{+}{}_{\dot\alpha\beta} = G^{+}{}_{\dot\alpha\beta}$$

$$\tilde{F}_0 = \frac{\mathrm{i}}{2p+1}F_0 - \frac{2\bar\mu^2}{2p+1}A + \frac{\mathrm{i}}{2(p+1)(2p+1)}\hat{M}^{\alpha\beta}G^{0}{}_{\alpha\beta}$$

$$\tilde{\psi}^{+}{}_{\dot\alpha+} = \psi^{+}{}_{\dot\alpha+} + \frac{4}{y\mu}(\sigma^\mu)^{+}{}_{\dot\alpha}{}^{\alpha}[\partial_\mu\alpha^{+}{}_{\alpha} + \partial_\mu\alpha^{-}{}_{\alpha}]$$

$$\tilde{\Psi}^{+}{}_{\dot\alpha-} = \tfrac{3}{2}\mathrm{i}\Omega^{+}{}_{\dot\alpha-} + \bar\mu^2\Psi^{+}{}_{\dot\alpha-}$$

$$\tilde{\gamma}^{+\pm}_{\dot\alpha\beta\alpha} = \gamma^{+\pm}_{\dot\alpha\beta\alpha} + \frac{1}{y\mu}(\sigma^\mu_{\pm})_{\alpha}{}^{\lambda}M^{+}_{+\dot\alpha\beta\lambda}{}^{\dot\delta}\partial_\mu\Psi^{+}{}_{\dot\delta-}$$

$$\tilde{\phi}^{\mp}{}_{\alpha 0} = \phi^{\mp}{}_{\alpha 0} + 2\mathrm{i}\bar\mu^2\alpha^{\mp}{}_{\alpha} + \frac{1}{2(p+1)}\hat{M}^{\alpha\beta}\gamma^{0\pm}_{\dot\alpha\beta\alpha} - \frac{2}{y\mu(p+1)}(\sigma^\mu_{\pm})_{\alpha}{}^{\alpha}\partial_\mu\Psi^{+}{}_{\dot\alpha-} - \frac{4}{y\mu}(\sigma^\mu_{\pm})_{\alpha}{}^{\dot\alpha}\partial_\mu\Psi^{-}{}_{\dot\alpha-}$$

$$\tilde{P}^{+\pm}_{\dot\alpha\alpha-} = \tfrac{3}{2}\mathrm{i}\, W^{+\pm}_{\dot\alpha\alpha-} + \bar\mu^2 P^{+\pm}_{\dot\alpha\alpha-} + \frac{4\mathrm{i}}{y\mu}(\sigma^\mu_{\pm})^{+}{}_{\dot\alpha\alpha}\partial_\mu F_-$$

$$\tilde{g}^{+}_{\dot\alpha\beta} = g^{+}{}_{\dot\alpha\beta} + \frac{1}{y\mu}(\sigma^\mu)^{\dot\lambda\alpha}M^{+}_{+\dot\alpha\beta\dot\lambda}{}^{\dot\delta}[\partial_\mu P^{++}_{\dot\delta\alpha-} + \partial_\mu P^{+-}_{\dot\delta\alpha-}]$$

**Table 2.** (continued)

$$\tilde{f}_0 = f_0 + 2\mathrm{i}\bar{\mu}^2 a + \frac{1}{2(p+1)}\hat{M}^{\alpha\beta} g^0{}_{\dot\alpha\dot\beta}$$
$$+\frac{2}{y\mu}(\sigma^\mu)^{\dot\alpha\alpha}\left(\frac{1}{p-1}\partial_\mu P^{+}_{\dot\alpha\alpha-} + \frac{1}{p+1}\partial_\mu P^{+-}_{\dot\alpha\alpha-} - \frac{2(p+1)}{p}\partial_\mu P^{-+}_{\dot\alpha\alpha-} - \frac{2(p+1)}{p}\partial_\mu P^{--}_{\dot\alpha\alpha-}\right)$$

$$\tilde{\psi}^+{}_{\dot\alpha-} = \tfrac{3}{2}\mathrm{i}\omega^+{}_{\alpha-} + \bar{\mu}^2\psi^+{}_{\dot\alpha-} + \frac{4\mathrm{i}}{y\mu}(\sigma^\mu)^+{}_\alpha{}^\alpha[\partial_\mu\phi^-{}_{\alpha-} + \partial_\mu\psi^-{}_{\alpha-}]$$

---

(*d*)

$$\tilde{\Psi}^-{}_{\dot\alpha+} = \Psi^-{}_{\dot\alpha+}$$

$$\tilde{P}_{\dot\alpha\alpha}{}^{-+}{}_+ = P^{-\pm}_{\dot\alpha\alpha+} + \frac{4}{y\mu}(\sigma^\mu_\pm)^-{}_{\alpha\alpha}\partial_\mu A$$

$$\tilde{G}^-{}_{\dot\alpha\beta} = G^-{}_{\dot\alpha\beta}$$

$$\tilde{G}^0{}_{\dot\alpha\dot\beta} = \frac{2\mathrm{i}(p+1)}{(2p+1)}G^0{}_{\alpha\beta} - \frac{\mathrm{i}}{2(2p+1)}\hat{M}_{\dot\alpha\dot\beta}F_0 + \frac{\bar{\mu}^2}{2(p+1)}\hat{M}_{\dot\alpha\beta}A$$

$$\tilde{\psi}^-{}_{\dot\alpha+} = \psi^-{}_{\dot\alpha+} + \frac{4}{y\mu}(\sigma^\mu)^-{}_{\dot\alpha}{}^\alpha[\partial_\mu\alpha^+{}_\alpha + \partial_\mu\alpha^-{}_\alpha]$$

$$\tilde{\Psi}^-{}_{\dot\alpha-} = \tfrac{3}{2}\mathrm{i}\Omega^-{}_{\dot\alpha-} + \bar{\mu}^2\Psi^-{}_{\dot\alpha-}$$

$$\tilde{\gamma}^{-\pm}_{\dot\alpha\dot\beta\alpha} = \gamma^{-\pm}_{\dot\alpha\dot\beta\alpha} + \frac{1}{y\mu}(\sigma^\mu_\pm)_\alpha{}^{\dot\lambda}M^-_{-\dot\alpha\beta\dot\lambda}{}^{\dot\delta}\partial_\mu\Psi^-{}_{\dot\delta-}$$

$$\tilde{\gamma}^{0\pm}_{\dot\alpha\beta\alpha} = 2(p+1)\gamma^{0\pm}_{\dot\alpha\beta\alpha} - \tfrac{1}{2}\hat{M}_{\dot\alpha\beta}\phi^\pm{}_{\alpha0} - \mathrm{i}\bar{\mu}^2\hat{M}_{\dot\alpha\beta}\alpha^\pm{}_\alpha + \frac{1}{y\mu}\hat{M}_{\dot\alpha\beta}(\sigma^\mu_\pm)_\alpha{}^{\dot\gamma}[2\partial_\mu\Psi^+{}_{\dot\gamma-} - (1/p)\partial_\mu\Psi^-{}_{\gamma--}]$$

$$\tilde{P}^{-\pm}_{\alpha\alpha-} = \tfrac{3}{2}\mathrm{i}\,W^{-\pm}_{\dot\alpha\alpha-} + \bar{\mu}^2P^{-\pm}_{\dot\alpha\alpha-} + \frac{4\mathrm{i}}{y\mu}(\sigma^\mu_\pm)^-{}_{\alpha\alpha}\partial_\mu F_-$$

$$\tilde{g}^-{}_{\dot\alpha\beta} = g^-{}_{\dot\alpha\beta} + \frac{1}{y\mu}(\sigma^\mu)^{\dot\lambda\alpha}M^-_{-\dot\alpha\beta\dot\lambda}{}^{\dot\delta}[\partial_\mu P^{-+}_{\dot\delta\alpha-} + \partial_\mu P^{--}_{\dot\delta\alpha-}]$$

$$g^0{}_{\dot\alpha\dot\beta} = 2(p+1)g^0{}_{\alpha\dot\beta} - \tfrac{1}{2}\hat{M}_{\alpha\beta}f_0 - \mathrm{i}\bar{\mu}^2\hat{M}_{\dot\alpha\beta}a$$
$$-\frac{1}{y\mu}(\sigma^\mu)^{\gamma\alpha}\hat{M}_{\alpha\beta}\left(\frac{2p}{p+1}\partial_\mu P^{++}_{\dot\gamma\alpha-} + \frac{2p}{p+1}\partial_\mu P^{++}_{\gamma\alpha-} + \frac{1}{p}\partial_\mu P^{-+}_{\gamma\alpha-} + \frac{1}{p}\partial_\mu P^{--}_{\dot\gamma\alpha-}\right)$$

$$\tilde{\psi}^-{}_{\dot\alpha-} = \tfrac{3}{2}\mathrm{i}\omega^-{}_{\alpha-} + \bar{\mu}^2\psi^-{}_{\alpha-} + \frac{4\mathrm{i}}{y\mu}(\sigma^\mu)^-{}_{\dot\alpha}{}^\alpha[\partial_\mu\phi^+{}_{\alpha-} + \partial_\mu\phi^-{}_{\alpha-}]$$

---

with monomials of $Q_{\alpha+}$ and $\bar{Q}_{\dot\alpha+}$ on $\tilde{\Psi}^\pm{}_{\dot\alpha+}$ and defining new fields as proportional to the Lorentz eigenstates projected from these variations. These bases are given in tables 2($c$) and 2($d$).

This procedure effectively provides a basis transformation of the superfield components into irreducible multiplets of $\mathscr{G}$. Such basis transformations may also be effected by constructing Casimirs of $\mathscr{G}$ which label different multiplets of $\mathscr{G}$ and finding functions of $\bar\theta^{\dot\alpha a}$ which form a complete set of eigenfunctions of this Casimir. Expanding the superfield in terms of these functions yields the appropriate basis directly as the component fields. Jarvis (1976) has used this technique for the study of unitary, irreducible representations of the $N=1$ super Poincaré algebra. Bufton and Taylor (1983) define similar basis functions for the $N$-extended supersymmetry algebras.

Given this new basis for the components of the superfield, the irreducibility of the multiplets we have generated can now be examined. It is found that the $F_+$ and $f_-$ multiplets are invariant subspaces while the $\Psi^+{}_{\dot\alpha+}$ and $\Psi^-{}_{\dot\alpha+}$ multiplets are invariant as factor spaces. This behaviour is typified by the following examples:

$$\delta_{Q_{\gamma+}}\tilde\omega^-_{\dot\alpha+} = -y\mu\tilde W^+_{\gamma\dot\alpha+} - y\mu\tilde W^{--}_{\gamma\dot\alpha+} - 4\mathrm{i}(\sigma^\mu)^-_{\gamma\dot\alpha}\partial_\mu\tilde f_+$$

$$\delta_{\bar Q_{\dot\gamma-}}\tilde a = -2\mathrm{i}\bar\mu^2\tilde\omega^+_{\dot\gamma-} - 2\mathrm{i}\bar\mu^2\tilde\omega^-_{\dot\gamma-}$$

$$\delta_{\bar Q_{\dot\gamma+}}\tilde F_0 = \frac{2\mathrm{i}}{(p+1)}\bar\mu^2\tilde\Psi^+{}_{\dot\gamma+} - \frac{\mathrm{i}}{(p+1)(2p+1)}\tilde\Omega^+{}_{\dot\gamma+} - \frac{2\mathrm{i}}{(2p+1)}\tilde\Omega^-{}_{\dot\gamma+}$$

**Table 3.** Basis required to obtain the irreducible 'spin-reducing' multiplets.

$$\hat\phi^\pm{}_{\alpha+} = \tilde\phi^\pm{}_{\alpha+} - \frac{1}{y\mu\bar\mu^2}(\sigma^\mu_\pm)_\alpha{}^{\dot\alpha}[\partial_\mu\tilde\Omega^+{}_{\dot\alpha+} + \partial_\mu\tilde\Omega^-{}_{\dot\alpha+}]$$

$$\hat f_+ = \tilde f_+ - \frac{1}{2y\mu\bar\mu^2}(\sigma^\mu)^{\dot\alpha\alpha}[\partial_\mu\tilde W^{++}_{\dot\alpha\alpha+} + \partial_\mu\tilde W^{+-}_{\dot\alpha\alpha+} + \partial_\mu W^{-+}_{\dot\alpha\alpha+} + \partial_\mu\tilde W^{--}_{\dot\alpha\alpha+}]$$

$$\hat W^{\pm\pm}_{\dot\alpha\alpha+} = \tilde W^{\pm\pm}_{\dot\alpha\alpha+} - \frac{2}{y\mu\bar\mu^2}(\sigma^\mu_\pm)^\pm{}_{\dot\alpha\alpha}\partial_\mu\tilde D$$

$$\hat\phi^\pm{}_{\alpha-} = \tilde\phi^\pm{}_{\alpha-} + \frac{1}{y\mu\bar\mu^2}(\sigma^\mu_\pm)_\alpha{}^{\dot\alpha}[\partial_\mu\tilde\omega^+{}_{\dot\alpha-} + \partial_\mu\tilde\omega^-{}_{\dot\alpha-}]$$

$$\hat F_- = \tilde F_- - \frac{1}{2y\mu\bar\mu^2}(\sigma^\mu)^{\dot\alpha\alpha}[\partial_\mu\tilde W^{++}_{\dot\alpha\alpha-} + \partial_\mu\tilde W^{+-}_{\dot\alpha\alpha-} + \partial_\mu\tilde W^{-+}_{\dot\alpha\alpha-} + \partial_\mu\tilde W^{--}_{\dot\alpha\alpha-}]$$

$$\hat W^{\pm\pm}_{\dot\alpha\alpha-} = \tilde W^{\pm\pm}_{\dot\alpha\alpha-} + \frac{2}{y\mu\bar\mu^2}(\sigma^\mu_\pm)^\pm{}_{\dot\alpha\alpha}\partial_\mu\tilde d$$

$$\hat P^{+\pm}_{\dot\alpha\alpha+} = \tilde P^{+\pm}_{\dot\alpha\alpha+} + \frac{1}{y\mu\bar\mu^2}(\sigma^\mu_\pm)_\alpha{}^\beta[2\mathrm{i}\partial_\mu\tilde G^+_{\beta\dot\alpha} + \tfrac{1}{2}\hat M_{\beta\dot\alpha}\partial_\mu\tilde F_0]$$

$$\hat\psi^+{}_{\dot\alpha+} = \tilde\psi^+{}_{\dot\alpha+} + \frac{\mathrm{i}}{2y\mu\bar\mu^2}(\sigma^\mu)^+{}_{\dot\alpha}{}^\alpha[\partial_\mu\tilde\phi^+{}_{\alpha0} + \partial_\mu\tilde\phi^-{}_{\alpha0}] + \frac{\mathrm{i}}{y\mu\bar\mu^2}(\sigma^\mu)^{\beta\alpha}[\partial_\mu\tilde\gamma^{++}_{\beta\dot\alpha\alpha} + \partial_\mu\tilde\gamma^{+-}_{\beta\dot\alpha\alpha}]$$

$$\hat\gamma^{+\pm}_{\dot\alpha\beta\alpha} = \tilde\gamma^{+\pm}_{\dot\alpha\beta\alpha} - \frac{1}{2y\mu\bar\mu^2}(\sigma^\mu_\pm)_\alpha{}^{\dot\lambda}M^+_{+\dot\alpha\beta\dot\lambda}{}^{\dot\delta}\partial_\mu\tilde\Psi^+{}_{\dot\delta-}$$

$$\hat\phi^\pm{}_{\alpha0} = \tilde\phi^\pm{}_{\alpha0} - \frac{(2p+1)}{(p+1)}\,\frac{1}{y\mu\bar\mu^2}(\sigma^\mu_\pm)_\alpha{}^{\dot\alpha}\partial_\mu\tilde\Psi^+{}_{\dot\alpha-}$$

$$\hat P^{-\pm}_{\alpha\dot\alpha+} = \tilde P^{-\pm}_{\dot\alpha\alpha+} + \frac{1}{y\mu\bar\mu^2}(\sigma^\mu_\pm)_\alpha{}^\beta[2\mathrm{i}\partial_\mu\tilde G^-{}_{\beta\dot\alpha} + \partial_\mu\tilde G^0{}_{\beta\dot\alpha}]$$

$$\hat\psi^-_{\dot\alpha+} = \tilde\psi^-{}_{\dot\alpha+} + \frac{\mathrm{i}}{2y\mu\bar\mu^2}(\sigma^\mu)^{\beta\mu}\left(\frac{1}{p+1}\partial_\mu\tilde\gamma^{0+}_{+\beta\dot\alpha\alpha} + \frac{1}{p+1}\partial_\mu\tilde\gamma^{0-}_{-\beta\dot\alpha\alpha} + 2\partial_\mu\tilde\gamma^{-+}_{\beta\dot\alpha\alpha} + 2\partial_\mu\tilde\gamma^{--}_{\beta\dot\alpha\alpha}\right)$$

$$\hat\gamma^{-\pm}_{\dot\alpha\beta\alpha} = \tilde\gamma^{-\pm}_{\dot\alpha\beta\alpha} - \frac{1}{2y\mu\bar\mu^2}(\sigma^\mu_\pm)_\alpha{}^{\dot\lambda}M^-_{-\dot\alpha\beta\dot\lambda}{}^{\dot\delta}\partial_\mu\tilde\Psi^-{}_{\dot\delta-}$$

$$\hat\gamma^{0\pm}_{\dot\alpha\beta\alpha} = \tilde\gamma^{0\pm}_{\dot\alpha\beta\alpha} + \frac{(2p+1)}{2p}\,\frac{1}{y\mu\bar\mu^2}(\sigma^\mu_\pm)_\alpha{}^{\dot\gamma}\hat M_{\dot\alpha\beta}\partial_\mu\tilde\Psi^-{}_{\dot\gamma-}$$

$$\delta_{\bar{Q}_{\gamma+}}\tilde{\psi}_{\dot{\alpha}} = 2i(\Pi^{-}\varepsilon)_{\dot{\alpha}\dot{\gamma}}\tilde{f}_{+} - \frac{4i}{y\mu}(\sigma^{\mu})_{\dot{\gamma}}{}^{\alpha}(\partial_{\mu}\tilde{P}^{-+}_{\alpha\dot{\alpha}+} + \partial_{\mu}\tilde{P}^{--}_{\alpha\dot{\alpha}+}).$$

As discussed in § 1, it has been pointed out (Fayet 1976, 1979, Sohnius 1978, Rands and Taylor 1983) that if the constraint

$$\bar{Z}Q_{\alpha\pm} = \pm i(\sigma^{\mu})_{\alpha}{}^{\dot{\alpha}}P_{\mu}\bar{Q}_{\dot{\alpha}\pm} \tag{16}$$

**Table 4.** Variations of the fields of the $\Lambda \equiv F_{+}$ multiplet under $Q_{\gamma+}$ and $\bar{Q}_{\dot{\lambda}+}$. These demonstrate the irreducibility, as a factor space, of the fields $F_{+}$, $\tilde{\Omega}^{+}{}_{\dot{\alpha}+}$, $\tilde{\Omega}^{-}{}_{\dot{\alpha}+}$, $\tilde{D}$ under the constraint $P^2+\mu^2\bar{\mu}^2=0$. $\delta_{\gamma+}\equiv\delta_{Q_{\gamma+}}$ and $\delta_{\dot{\lambda}+}\equiv\delta_{\bar{Q}_{\dot{\lambda}+}}$, and we recall $P^2=-\partial^2$, $\mu^2\equiv Z$ and $\bar{\mu}^2\equiv\bar{Z}$.

$$\delta_{\gamma+}F_{+} = \delta_{\dot{\lambda}+}F_{+} = 0$$

$$\delta_{\gamma+}\hat{\phi}^{\pm}{}_{\alpha+} = \frac{y}{\mu\bar{\mu}^2}(\Pi^{\pm}\varepsilon)_{\alpha\gamma}(P^2+\mu^2\bar{\mu}^2)F_{+}$$

$$\delta_{\dot{\lambda}+}\hat{\phi}^{\pm}{}_{\alpha+} = 0$$

$$\delta_{\gamma+}\tilde{\Omega}^{\pm}{}_{\dot{\alpha}+} = -4i(\sigma^{\mu})^{\pm}{}_{\gamma\dot{\alpha}}\partial_{\mu}F_{+}$$

$$\delta_{\dot{\lambda}+}\tilde{\Omega}^{\pm}{}_{\dot{\alpha}+} = -4i\bar{\mu}^2(\Pi^{\pm}\varepsilon)_{\dot{\alpha}\dot{\lambda}}F_{+}$$

$$\delta_{\gamma+}\hat{f}_{+} = \frac{y}{\mu\bar{\mu}^2}(\tfrac{1}{2}P^2+\mu^2\bar{\mu}^2)(\phi^{+}{}_{\gamma+}+\phi^{-}{}_{\gamma+}) + \frac{2i}{y^2\mu^2\bar{\mu}^4}(\sigma^{\mu})^{\dot{\alpha}}{}_{\gamma}(P^2+\mu^2\bar{\mu}^2)(\partial_{\mu}\tilde{\Psi}^{+}{}_{\dot{\alpha}+}+\partial_{\mu}\tilde{\Psi}^{-}{}_{\dot{\alpha}+})$$

$$\delta_{\dot{\lambda}+}\hat{f}_{+} = \frac{2i}{y\mu}(\sigma^{\mu})_{\dot{\lambda}}{}^{\alpha}(\partial_{\mu}\hat{\phi}^{+}{}_{\alpha+}+\partial_{\mu}\hat{\phi}^{-}{}_{\alpha+})$$

$$\delta_{\gamma+}\tilde{D} = -2i(\sigma^{\mu})^{\dot{\alpha}}{}_{\gamma}(\partial_{\mu}\tilde{\Omega}^{+}{}_{\dot{\alpha}+}+\partial_{\mu}\tilde{\Omega}^{-}{}_{\dot{\alpha}+})$$

$$\delta_{\dot{\lambda}+}\tilde{D} = -2i\bar{\mu}^2(\tilde{\Omega}^{+}{}_{\dot{\lambda}+}+\tilde{\Omega}^{-}{}_{\dot{\lambda}+})$$

$$\delta_{\gamma+}\hat{W}^{\pm\pm}_{\dot{\alpha}\alpha+} = -4i(\sigma^{\mu})^{\pm}{}_{\dot{\alpha}\gamma}\partial_{\mu}\hat{\phi}^{\pm}{}_{\alpha+} - \frac{y}{\mu\bar{\mu}^2}(\Pi^{\pm}\varepsilon)_{\alpha\gamma}(P^2+\mu^2\bar{\mu}^2)\tilde{\Omega}^{\pm}{}_{\alpha+}$$

$$\delta_{\dot{\lambda}+}\hat{W}^{\pm\pm}_{\dot{\alpha}\alpha+} = -4i\bar{\mu}^2(\Pi^{\pm}\varepsilon)_{\dot{\alpha}\dot{\lambda}}\hat{\phi}^{\pm}{}_{\alpha+}$$

$$\delta_{\gamma+}\tilde{\delta}^{\pm}{}_{\alpha} = \frac{y}{\mu\bar{\mu}^2}(\Pi^{\pm}\varepsilon)_{\alpha\gamma}(P^2+\mu^2\bar{\mu}^2)\tilde{D} - 2i(\sigma^{\mu})^{\dot{\alpha}}{}_{\gamma}(\partial_{\mu}\hat{W}^{+\pm}_{\dot{\alpha}\alpha+}+\partial_{\mu}\hat{W}^{-\pm}_{\dot{\alpha}\alpha+})$$

$$\delta_{\dot{\lambda}+}\tilde{\delta}^{\pm}{}_{\alpha} = -2i\bar{\mu}^2(\hat{W}^{+\pm}_{\dot{\lambda}\alpha+}+\hat{W}^{-\pm}_{\dot{\lambda}\alpha+})$$

$$\begin{aligned}\delta_{\gamma+}\tilde{\omega}^{\pm}{}_{\dot{\alpha}+} = {}& -4i(\sigma^{\mu})^{\pm}{}_{\dot{\alpha}\gamma}\partial_{\mu}\hat{f}_{+} - y\mu(\hat{W}^{\pm+}_{\dot{\alpha}\gamma+}+W^{\pm-}_{\dot{\alpha}\gamma+})\\ & -\frac{2i}{y\mu\bar{\mu}^2}(\sigma^{\mu})^{\pm}{}_{\dot{\alpha}\gamma}(\sigma^{\nu})^{\beta\dot{\beta}}\partial_{\mu}\partial_{\nu}(\hat{W}^{++}_{\dot{\beta}\beta+}+\hat{W}^{+-}_{\dot{\beta}\beta+}+\hat{W}^{-+}_{\dot{\beta}\beta+}+\hat{W}^{--}_{\dot{\beta}\beta+})\\ & -\frac{2}{\mu^2\bar{\mu}^4}(\sigma^{\mu})^{\pm}{}_{\dot{\alpha}\gamma}(P^2+\mu^2\bar{\mu}^2)\partial_{\mu}\tilde{D}\end{aligned}$$

$$\begin{aligned}\delta_{\dot{\lambda}+}\tilde{\omega}^{\pm}{}_{\dot{\alpha}+} = {}& -4i\bar{\mu}^2(\Pi^{\pm}\varepsilon)_{\dot{\alpha}\dot{\lambda}}\hat{f}_{+} - \frac{2i}{y\mu}(\sigma^{\mu})_{\dot{\lambda}}{}^{\alpha}\partial_{\mu}(\hat{W}^{\pm+}_{\dot{\alpha}\alpha+}+\hat{W}^{\pm-}_{\dot{\alpha}\alpha+})\\ & -\frac{2i}{y\mu}(\sigma^{\mu})^{\pm\alpha}_{\dot{\alpha}}\partial_{\mu}(\hat{W}^{++}_{\dot{\lambda}\alpha+}+\hat{W}^{+-}_{\dot{\lambda}\alpha+}+\hat{W}^{-+}_{\dot{\lambda}\alpha+}+\hat{W}^{--}_{\dot{\lambda}\alpha+})\end{aligned}$$

$$\delta_{\dot{\lambda}+}\tilde{d} = -2i\bar{\mu}^2(\tilde{\omega}^{+}{}_{\dot{\lambda}+}+\tilde{\omega}^{-}{}_{\dot{\lambda}+}) + \frac{y}{\mu}(\sigma^{\mu})_{\dot{\lambda}}{}^{\alpha}(\partial_{\mu}\tilde{\delta}^{+}{}_{\alpha}+\partial_{\mu}\tilde{\delta}^{-}{}_{\alpha})$$

$$\delta_{\gamma+}\tilde{d} = -2i(\sigma^{\mu})^{\dot{\alpha}}{}_{\gamma}(\partial_{\mu}\tilde{\omega}^{+}{}_{\dot{\alpha}+}+\partial_{\mu}\tilde{\omega}^{-}{}_{\dot{\alpha}+}) + y\mu(\tilde{\delta}^{+}{}_{\gamma}+\tilde{\delta}^{-}{}_{\gamma})$$

is imposed, the number of fields in an irreducible representation is reduced from $2^4$ to $2^2$. This constraint implies that

$$P^2 + Z\bar{Z} = 0. \tag{17}$$

In the present approach this reduction takes place via the imposition of only the weaker constraint (17) as described below.

To observe this phenomenon we note the intimate connection between (16) and (17) and use this to introduce further field redefinitions, for the fields in each multiplet which are obtained from acting with $Q_{\alpha-}$, $Q_{\alpha-}Q_{\beta-}$ and $Q_{\alpha-}\bar{Q}_{\dot{\alpha}-}$ on the highest weight state of the multiplet or with $Q_{\alpha+}$, $Q_{\alpha+}Q_{\beta+}$, $Q_{\alpha+}\bar{Q}_{\dot{\alpha}+}$ on the lowest weight state of the multiplet. These fields are constructed, up to a proportionality, from the ~ basis of table 2 by projecting Lorentz eigenstates either from

$$(Q_{\alpha-} + (\mathrm{i}/\bar{Z})(\sigma^{\mu})_{\alpha}{}^{\dot{\alpha}} P_{\mu} \bar{Q}_{\dot{\alpha}-})\tilde{B} \tag{18}$$

where $\tilde{B}$ is the generic title given to the fields obtained from $\Lambda_i$, $Q_{\alpha-}\Lambda_i$ and $\bar{Q}_{\dot{\alpha}-}\Lambda_i$ with $\Lambda_i$ the highest weight state of a multiplet, or from,

$$(Q_{\alpha+} - (\mathrm{i}/Z)(\sigma^{\mu})_{\alpha}{}^{\dot{\alpha}} P_{\mu} \bar{Q}_{\dot{\alpha}+})\tilde{B} \tag{19}$$

where $\tilde{B}$ refers here to the fields obtained from $\bar{\Lambda}$, $Q_{\alpha+}\bar{\Lambda}$ and $\bar{Q}_{\dot{\alpha}+}\bar{\Lambda}$ wth $\bar{\Lambda}$ the lowest weight state of a multiplet. These field redefinitions are given in table 3 and in this basis it is observed that the fields $\Lambda_i$, $\bar{Q}_{\dot{\alpha}\pm}\Lambda_i$ and $\bar{Q}_{\dot{\alpha}\pm}\bar{Q}_{\dot{\beta}\pm}\Lambda_i$ (taking upper (lower) signs if $\Lambda_i$ is a lowest (highest) weight vector) are invariant as a factor space with the remaining fields of each multiplet decoupling when $P^2 + Z\bar{Z} = 0$. This is demonstrated for the $F_+$ multiplet in table 4, which clearly shows that when condition (17) is imposed, an irreducible realisation of the SO(2)-extended super Poincaré algebra consists of four fields with $\mathrm{O}(3,1) \times \mathrm{U}(1)$ labels:

$$\{(p, q, T), (p+\tfrac{1}{2}, q, T-1), (p-\tfrac{1}{2}, q, T-1), (p, q, T-2)\}.$$

## 3. Conclusion

In this paper new realisations of centrally extended $N$-supersymmetry have been constructed with $4^{N+[N/2]}$ rather than $16^N$ component fields. The emphasis has been on establishing the representations in a general framework for arbitrary external 'superspin' labels, rather than on investigating the key questions of whether these afford dynamical models which avoid some of the usual difficulties occurring in quantised supersymmetric theories with central charges. Future studies along these lines will involve low-spin superfields of the general class treated here.

## Appendix 1. Projection operators for spin $M \times \frac{1}{2}$ and spin $M \times 1$

Section 2 requires the use of spin $M \times \frac{1}{2}$ and $M \times 1$, with respect to SU(2) projection operators (cf Farmer and Jarvis 1983). The two-index basis for SU(2) is related to the spherical basis via

$$\hat{M}_{\alpha\beta} = 2(\hat{\boldsymbol{M}} \cdot \boldsymbol{\sigma}\varepsilon)_{\alpha\beta} \tag{A1.1}$$

where the generators are in a spin $M$ matrix representation of SU(2). Where these act on superfield components such as $\psi_{\alpha}$ or $G_{\alpha\beta}$, the question arises of projections onto total spins $(M\pm\frac{1}{2})$ or $(M, M\pm 1)$, respectively. These are derived using the characteristic identity (quadratic or cubic respectively) satisfied by the generators in the reducible $M\times\frac{1}{2}$ and $M\times 1$ representations.

The general construction of projection operators proceeds as follows. Consider some reducible representation of an algebra with Casimir operator, $C$, and eigenvalues $c_1, c_2, \ldots, c_n$. Then there exists a complete set of projection operators

$$\Pi_i=\prod_{\substack{j=1\\ j\neq 1}}^{n}\frac{(C-c_j)}{(c_i-c_j)} \tag{A1.2}$$

such that $\Pi_i\cdot\Pi_j=\delta_{ij}\Pi_i$ and $\Sigma_{i=1}^{n}\Pi_i=1$. Each of the $\Pi_i$ will extract a subspace with eigenvalue $c_i$ with respect to $C$.

For $M\times\frac{1}{2}$ we have for the Casimir (spin $M$ indices are suppressed and indices $\alpha, \beta, \ldots$ are raised using the inverse metric $\varepsilon^{\alpha\beta}$)

$$(\hat{\boldsymbol{M}}\cdot\boldsymbol{\sigma})_\alpha{}^\beta=(\hat{\boldsymbol{M}}+\tfrac{1}{2}\boldsymbol{\sigma})^2{}_\alpha{}^\beta-(\hat{\boldsymbol{M}})^2\delta_\alpha^\beta-(\tfrac{1}{2}\boldsymbol{\sigma})^2{}_\alpha{}^\beta \tag{A1.3}$$

where $\hat{\boldsymbol{M}}$ and $\frac{1}{2}\boldsymbol{\sigma}$ are spin $M$ and spin $\frac{1}{2}$ matrix representations respectively. The eigenvalues of $(\hat{\boldsymbol{M}}\cdot\boldsymbol{\sigma})_\alpha{}^\beta$ on the reducible $M\times\frac{1}{2}$ space are given by

$$(M\pm\tfrac{1}{2})\text{ subspace:}\quad (M\pm\tfrac{1}{2})(M\pm\tfrac{1}{2}+1)-M(M+1)-\tfrac{1}{2}(\tfrac{1}{2}+1)=M^\pm \tag{A1.4}$$

where $M^+=M$ and $M^-=-M-1$. The projection operators are therefore given by

$$\Pi_\alpha^{\pm\frac{1}{2}\beta}=(\hat{M}_\alpha{}^\beta-2M^\mp\delta_\alpha^\beta)/2(2M^\pm+1) \tag{A1.5}$$

where (A1.1) has been used. The following expressions can easily be derived from (A1.5) and are frequently used:

$$\delta_\alpha^\beta=\Pi^{+\frac{1}{2}\beta}_{\ \alpha}+\Pi^{-\frac{1}{2}\beta}_{\ \alpha} \tag{A1.6}$$

$$\hat{M}_\alpha{}^\beta=2M^+\Pi^{+\frac{1}{2}\beta}_{\ \alpha}+2M^-\Pi^{-\frac{1}{2}\beta}_{\ \alpha} \tag{A1.7}$$

$$\hat{M}_\alpha{}^\beta\hat{M}_\beta{}^\gamma=4M(M+1)\delta_\alpha^\gamma-2\hat{M}_\alpha{}^\gamma. \tag{A1.8}$$

For $M\times 1$ we have the Casimir

$$(\hat{\boldsymbol{M}}\cdot\boldsymbol{\Sigma})_{\alpha\beta}^{\gamma\delta}=(\hat{\boldsymbol{M}}+\tfrac{1}{2}\boldsymbol{\Sigma})^{2\gamma\delta}_{\ \alpha\beta}-(\hat{\boldsymbol{M}})^2 1^{\gamma\delta}_{\alpha\beta}-(\tfrac{1}{2}\boldsymbol{\Sigma})^{2\gamma\delta}_{\ \alpha\beta} \tag{A1.9}$$

where

$$\tfrac{1}{2}\boldsymbol{\Sigma}^{\gamma\delta}_{\alpha\beta}=\tfrac{1}{2}(\boldsymbol{\sigma}_\alpha{}^\gamma\delta_\beta{}^\delta+\delta_\alpha{}^\gamma\boldsymbol{\sigma}_\beta{}^\delta+\boldsymbol{\sigma}_\beta{}^\gamma\delta_\alpha{}^\delta+\delta_\beta{}^\gamma\boldsymbol{\sigma}_\alpha{}^\delta) \tag{A1.10}$$

is the spin-1 matrix representation and

$$1^{\gamma\delta}_{\alpha\beta}=\tfrac{1}{2}(\delta_\alpha{}^\gamma\delta_\beta{}^\delta+\delta_\alpha{}^\delta\delta_\beta{}^\gamma). \tag{A1.11}$$

The eigenvalues of $(\hat{\boldsymbol{M}}\cdot\boldsymbol{\Sigma})^{\gamma\delta}_{\alpha\beta}$ on the reducible $M\times 1$ space are given by

$$(M\pm 1)\text{ subspace:}\quad (M\pm 1)(M\pm 1+1)-M(M+1)-1(1+1)=2M^\pm \tag{A1.12}$$

$$(M)\text{ subspace:}\quad M(M+1)-M(M+1)-1(1+1)=-2. \tag{A1.13}$$

Thus the projection operators are

$$\Pi^{\pm 1\gamma\delta}_{\ \alpha\beta}=\left(\frac{\hat{N}+(2M^\pm+3)\hat{L}+4(M^\pm+1)(M^\pm+2)1}{8(M^\pm+1)(2M^\pm+1)}\right)^{\gamma\delta}_{\alpha\beta} \tag{A1.14}$$

$$\Pi^{0\gamma\delta}_{\alpha\beta}=\left(\frac{\hat{N}+\hat{L}+4M^+M^-1}{8M^+M^-}\right)^{\gamma\delta}_{\alpha\beta} \tag{A1.15}$$

where we have used (A1.8) and the following definitions

$$\hat{L}^{\gamma\delta}_{\alpha\beta}=\tfrac{1}{2}(\hat{M}^{\gamma}_{\alpha}\delta^{\delta}_{\beta}+\delta^{\gamma}_{\alpha}\hat{M}^{\delta}_{\beta}+\hat{M}^{\gamma}_{\beta}\delta^{\delta}_{\alpha}+\delta^{\gamma}_{\beta}\hat{M}^{\delta}_{\alpha}) \tag{A1.16}$$

$$\hat{N}^{\gamma\delta}_{\alpha\beta}=\tfrac{1}{4}(\hat{M}^{\gamma}_{\alpha}\hat{M}^{\delta}_{\beta}+\hat{M}^{\delta}_{\alpha}\hat{M}^{\gamma}_{\beta}+\hat{M}^{\gamma}_{\beta}\hat{M}^{\delta}_{\alpha}+\hat{M}^{\delta}_{\beta}\hat{M}^{\gamma}_{\alpha}). \tag{A1.17}$$

From these definitions several useful identities can be derived which are necessary for the extraction of component field variations. Examples are:

$$\Pi^{\pm\frac{1}{2}\beta}_{\ \alpha}\Pi^{0}{}_{\lambda\beta}^{\ \gamma\delta}=\pm\frac{M^{\pm}}{2M+1}\left(\Pi^{0}{}_{\lambda\alpha}^{\ \gamma\delta}+\frac{1}{2M^{\pm}}\varepsilon_{\alpha\lambda}\hat{M}^{\gamma\delta}\right)$$

$$\Pi^{0}{}_{\alpha\beta}^{\ \gamma\delta}\Pi^{\pm\frac{1}{2}\lambda}_{\ \gamma}=\pm\frac{M^{\pm}}{2M+1}\left(\Pi^{0}{}_{\alpha\beta}^{\ \delta\lambda}+\frac{1}{2M^{\pm}}\varepsilon^{\delta\lambda}\hat{M}_{\alpha\beta}\right)$$

$$\Pi^{\pm\frac{1}{2}\beta}_{\ \alpha}\Pi^{\pm1\gamma\delta}_{\ \lambda\beta}=\Pi^{\pm1\gamma\delta}_{\ \lambda\alpha}=\Pi^{\pm1\gamma\beta}_{\ \lambda\alpha}\Pi^{\pm\frac{1}{2}\delta}_{\ \beta}$$

$$\Pi^{\pm\frac{1}{2}\beta}_{\ \alpha}\Pi^{\mp1\gamma\delta}_{\ \lambda\beta}=0=\Pi^{\mp1\gamma\beta}_{\ \lambda\alpha}\Pi^{\pm\frac{1}{2}\delta}_{\ \beta}$$

$$\Pi^{0}{}_{\alpha\beta}^{\ \delta\rho}\varepsilon_{\gamma\rho}\psi^{\pm}_{\delta}=-\frac{1}{2M^{\mp}}\hat{M}_{\alpha\beta}\psi^{\pm}_{\gamma}$$

$$\Pi^{\pm1\delta\rho}_{\ \alpha\beta}\varepsilon_{\gamma\rho}\psi^{\pm}_{\delta}=\left(\frac{1}{2M^{\mp}}\hat{M}_{\alpha\beta}\delta_{\gamma}{}^{\delta}+\tfrac{1}{2}\varepsilon_{\gamma\alpha}\delta_{\beta}{}^{\delta}+\tfrac{1}{2}\varepsilon_{\gamma\beta}\delta_{\alpha}{}^{\delta}\right)\psi^{\pm}_{\delta}$$

$$\Pi^{\pm1\delta\rho}_{\ \alpha\beta}\varepsilon_{\gamma\rho}\psi^{\mp}_{\delta}=0$$

$$\Pi^{\pm\frac{1}{2}\beta}_{\ \gamma}\hat{M}_{\alpha\beta}=M^{\pm}\Pi^{\pm\frac{1}{2}\rho}_{\ \gamma}\varepsilon_{\beta\alpha}$$

$$\Pi^{\pm1\gamma\delta}_{\ \alpha\beta}\hat{M}_{\gamma\delta}=0=\hat{M}^{\alpha\beta}\Pi^{\pm1\gamma\beta}_{\ \alpha\beta}$$

$$\Pi^{0}{}_{\alpha\beta}^{\ \gamma\delta}\hat{M}_{\gamma\delta}=\hat{M}_{\alpha\beta}$$

$$\hat{M}^{\alpha\beta}\Pi^{0}{}_{\alpha\beta}^{\ \gamma\delta}=\hat{M}^{\gamma\delta}.$$

## Appendix 2. $\bar{\theta}$ conventions and some useful identities

Conventions:

$$\varepsilon^{\dot{\alpha}\dot{\beta}}=\varepsilon^{ab}=-\varepsilon_{\dot{\alpha}\dot{\beta}}=-\varepsilon_{ab}$$

where $\dot{\alpha}, \dot{\beta}=1,2$, $a, b=+,-$ and $\varepsilon^{12}=\varepsilon^{+-}=+1$.

$$\varepsilon^{\dot{\alpha}\dot{\beta}}\varepsilon_{\dot{\beta}\dot{\gamma}}=\delta^{\dot{\alpha}}_{\dot{\gamma}} \qquad \varepsilon^{ab}\varepsilon_{bc}=\delta^{a}_{c}$$

$$\bar{\theta}_{\dot{\alpha}a}=\varepsilon_{\dot{\alpha}\dot{\beta}}\varepsilon_{ba}\bar{\theta}^{\dot{\beta}b} \qquad \bar{\theta}^{\dot{\alpha}a}=\varepsilon^{\dot{\alpha}\dot{\beta}}\varepsilon^{ba}\bar{\theta}_{\dot{\beta}b}.$$

Metric $\eta_{\mu\nu}=(-,+,+,+)$

$$\sigma^{\mu}=(1,\sigma^{i}) \qquad \bar{\sigma}^{\mu}=(1,-\sigma^{i}) \qquad \mathrm{Tr}\,\sigma^{\mu}\bar{\sigma}^{\nu}=-2\eta^{\mu\nu}.$$

The monomial bases for $\bar{\theta}^{\dot{\alpha}a}$ expansions in § 2 are given below together with some useful identities associated with taking products and derivatives.

Calculus:

$$\partial_{\dot{\alpha}a}\bar{\theta}^{\dot{\beta}b}=\delta_{\dot{\alpha}}{}^{\dot{\beta}}\delta_{a}{}^{b}$$

$$\delta_{\dot{\alpha}a}(\bar{\theta}\bar{\theta})^{bc}=\delta_{a}{}^{b}\bar{\theta}_{\dot{\alpha}}{}^{c}+\delta_{a}{}^{c}\bar{\theta}_{\dot{\alpha}}{}^{b}$$

$$\partial_{\dot\alpha a}(\bar\theta\bar\theta)^{\dot\beta\dot\gamma}=\delta_{\dot\alpha}{}^{\dot\beta}\bar\theta^{\dot\gamma}{}_a+\delta_{\dot\alpha}{}^{\dot\gamma}\bar\theta^{\dot\beta}{}_a$$

$$\partial_{\dot\alpha a}(\bar\theta^3)^{\dot\beta b}=\tfrac{3}{2}(\bar\theta\bar\theta)^b{}_a\delta_{\dot\alpha}{}^{\dot\beta}+\tfrac{3}{2}(\bar\theta\bar\theta)^{\dot\beta}{}_{\dot\alpha}\delta_a{}^b$$

$$\partial_{\dot\alpha a}(\bar\theta^4)=-4(\bar\theta^3)_{\dot\alpha a}.$$

Identities:

$$(\bar\theta\bar\theta)^{ab}=\bar\theta^{\dot\alpha a}\bar\theta_{\dot\alpha}{}^b$$

$$(\bar\theta\bar\theta)^{\dot\alpha\dot\beta}=\bar\theta^{\dot\alpha a}\bar\theta^{\dot\beta}{}_a$$

$$(\bar\theta^3)^{\dot\alpha a}=(\bar\theta\bar\theta)^{\dot\alpha\dot\beta}\bar\theta_{\dot\beta}{}^a=-(\bar\theta\bar\theta)^{ab}\bar\theta^{\dot\alpha}{}_b$$

$$(\bar\theta^4)=(\bar\theta^3)^{\dot\alpha a}\bar\theta_{\dot\alpha a}=(\bar\theta\bar\theta)^{\dot\alpha\dot\beta}(\bar\theta\bar\theta)_{\dot\alpha\dot\beta}=-(\bar\theta\bar\theta)^{ab}(\bar\theta\bar\theta)_{ab}$$

$$\bar\theta^{\dot\alpha a}\bar\theta^{\dot\beta}{}_b=\tfrac{1}{2}(\bar\theta\bar\theta)^{\dot\alpha\dot\beta}\delta^a_b-\tfrac{1}{2}(\bar\theta\bar\theta)^a{}_b\varepsilon^{\dot\alpha\dot\beta}$$

$$\bar\theta^{\dot\alpha}{}_a(\bar\theta\bar\theta)^{bc}=-\tfrac{1}{3}(\bar\theta^3)^{\dot\alpha b}\delta^c_a-\tfrac{1}{3}(\bar\theta^3)^{\dot\alpha c}\delta^b_a$$

$$\bar\theta^{\dot\alpha}{}_a(\bar\theta\bar\theta)^{\dot\beta\dot\gamma}=\tfrac{1}{3}(\bar\theta^3)^{\dot\beta}{}_a\varepsilon^{\dot\alpha\dot\gamma}+\tfrac{1}{3}(\bar\theta^3)^{\dot\gamma}{}_a\varepsilon^{\dot\alpha\dot\beta}$$

$$\bar\theta^{\dot\alpha}{}_a(\bar\theta^3)^{\dot\beta b}=-\tfrac{1}{4}(\bar\theta^4)\varepsilon^{\dot\alpha\dot\beta}\delta^b_a$$

$$(\bar\theta\bar\theta)^{\dot\alpha\dot\beta}(\bar\theta\bar\theta)^{ab}=0$$

$$(\bar\theta\bar\theta)^{\dot\alpha\dot\beta}(\bar\theta\bar\theta)^{\dot\gamma\dot\delta}=\tfrac{1}{6}(\varepsilon^{\dot\beta\dot\gamma}\varepsilon^{\dot\alpha\dot\delta}+\varepsilon^{\dot\alpha\dot\gamma}\varepsilon^{\dot\beta\dot\delta})(\bar\theta^4)$$

$$(\bar\theta\bar\theta)^{ab}(\bar\theta\bar\theta)^{cd}=\tfrac{1}{6}(\varepsilon^{bc}\varepsilon^{ad}+\varepsilon^{ac}\varepsilon^{bd})(\bar\theta^4).$$

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